Modeling Fourier Amplitude Spectra of Earthquake Ground Motions

Jeff Bayless
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With thanks to my PhD advisor
Norman Abrahamson

Dr Jeff Bayless

Engineering Seismologist
AECOM, Los Angeles

jeff.bayless@aecom.com
www.jeff-bayless.com

Biography

PhD: University of California, Davis
MS, BS: University of California, Los Angeles

Mentors Paul Somerville, Norman Abrahamson, Christine Goulet

Experience in practice is mostly with ground shaking seismic hazard

Research interests include all types of ground motion studies: near-field effects, attenuation, simulations, site response, etc.

Introduction: Response spectra vs. Fourier spectra

Ground Motion Model for FAS

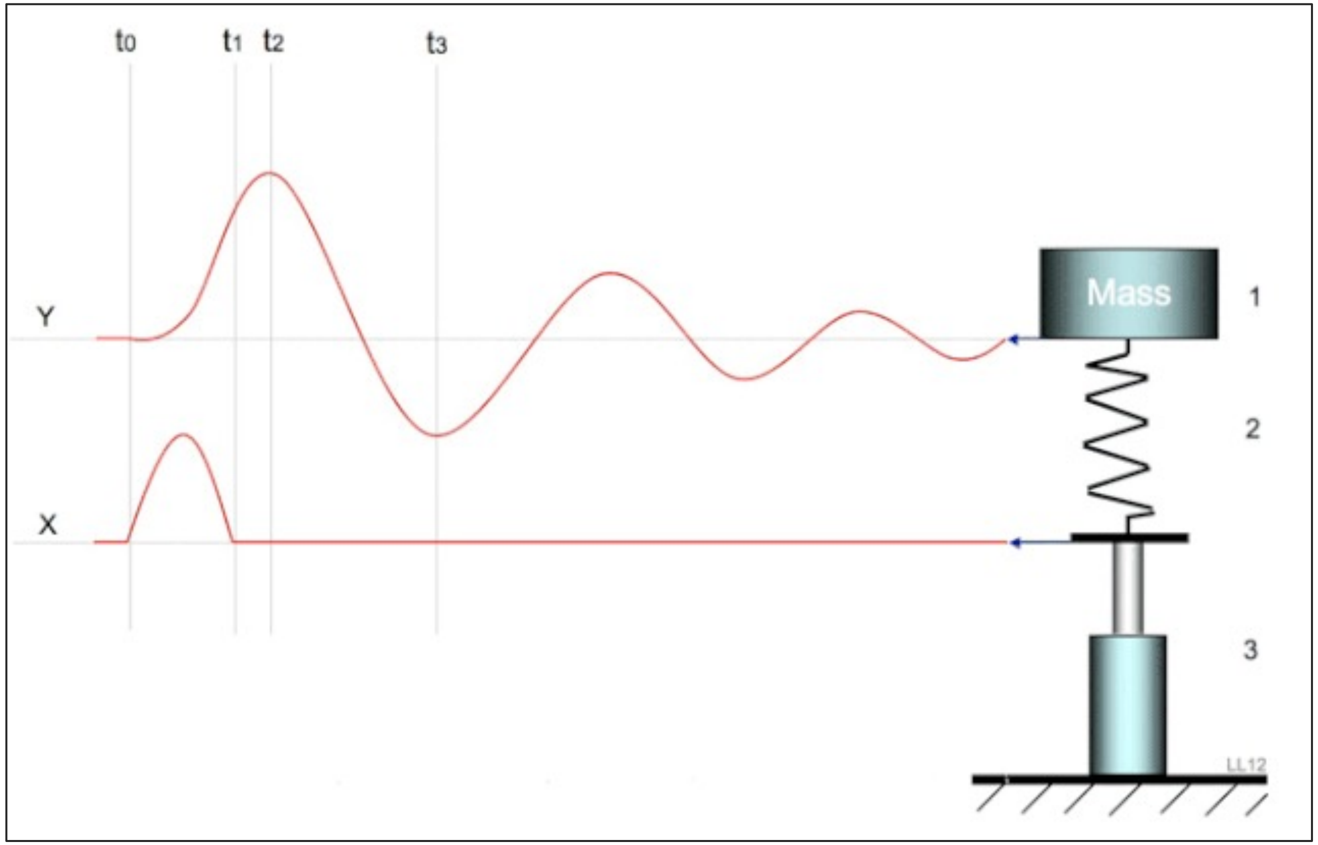
Inter-frequency correlation of the FAS

Introduction

Response Spectrum

- Is a measure of peak responses of a series of SDOF oscillators different fundamental periods.
- Ubiquitous intensity measure (IM) in earthquake engineering.
- **Because of the SDOF structure, does not directly represent the ground motion itself.**
- **Because it is the peak of the response in time, the response spectrum is not a linear operation.**

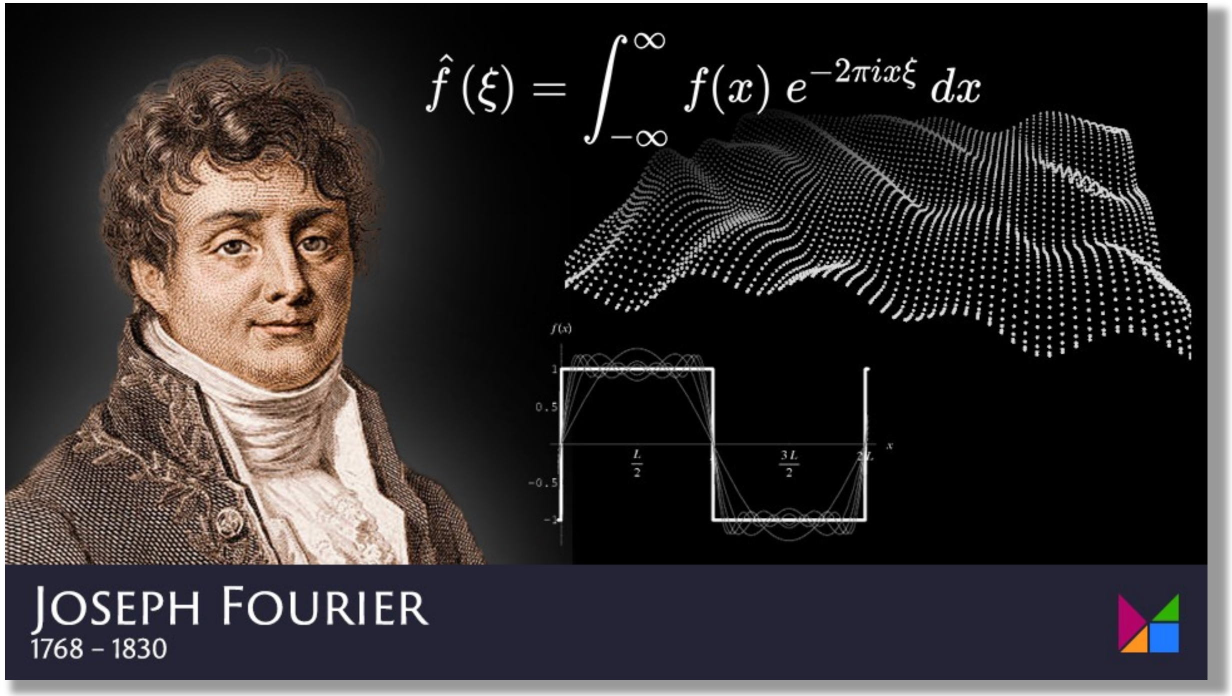
https://commons.wikimedia.org/wiki/File:SR-SDoF.gif



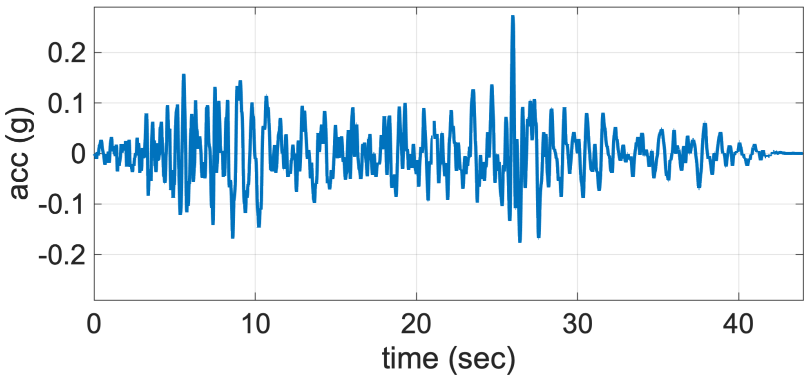
Introduction

Fourier amplitude and phase spectra

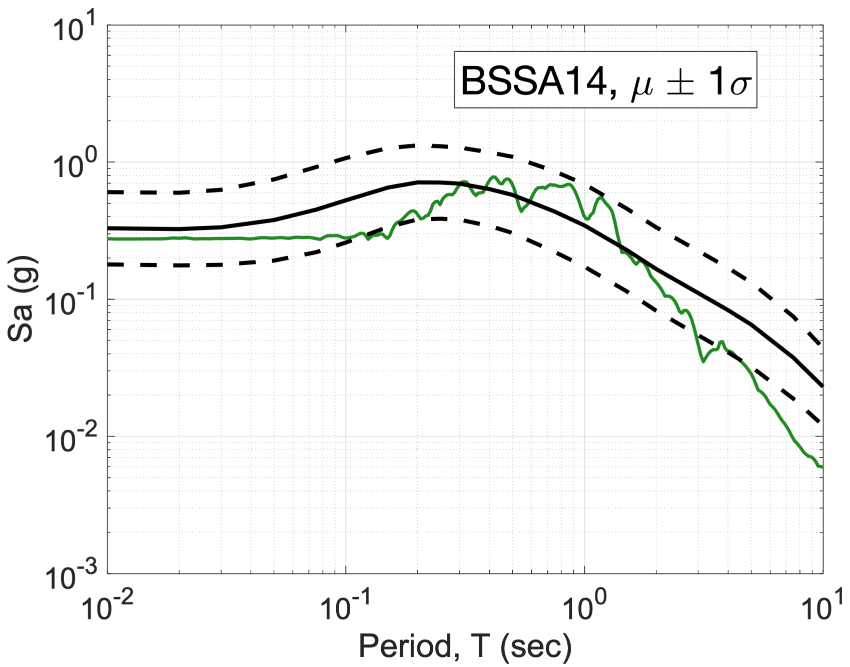
- The Fourier Transform is a mathematical model which transforms signals between two domains – e.g. time and frequency.
- Decomposes the signal into sine and cosine components, describing the frequencies present.
- A direct representation of the earthquake ground motion.
- **The Fourier Transform is a linear operation.**



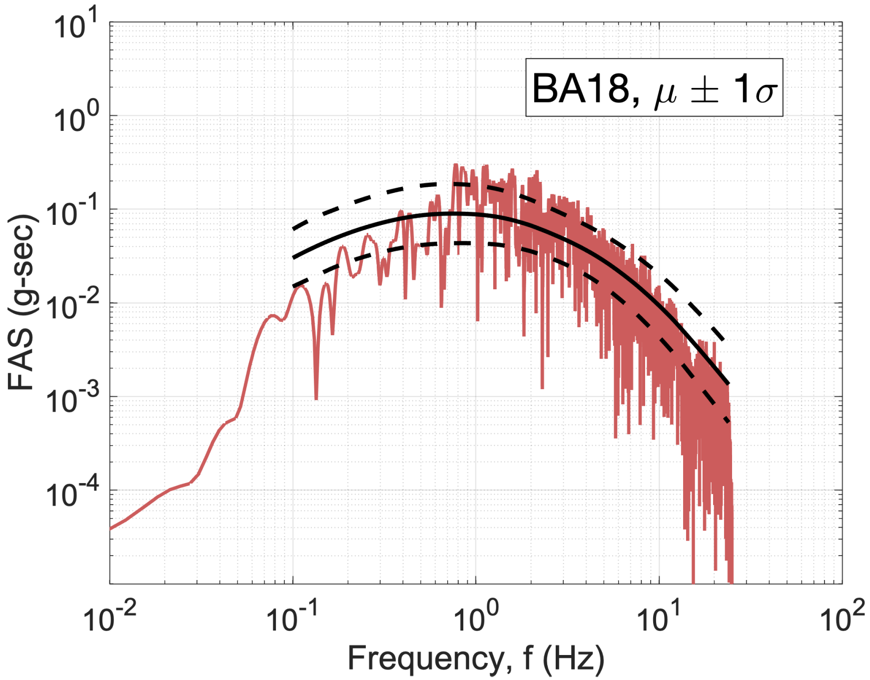
Introduction



Response spectrum (Sa)



Fourier amplitude spectrum (FAS)



Introduction

The response spectrum at short periods primarily reflects the strength of the underlying Fourier spectral amplitudes over a broad range of frequencies.

SDOF response (linear DE)

$$u(t) = h(t) \otimes p(t)$$

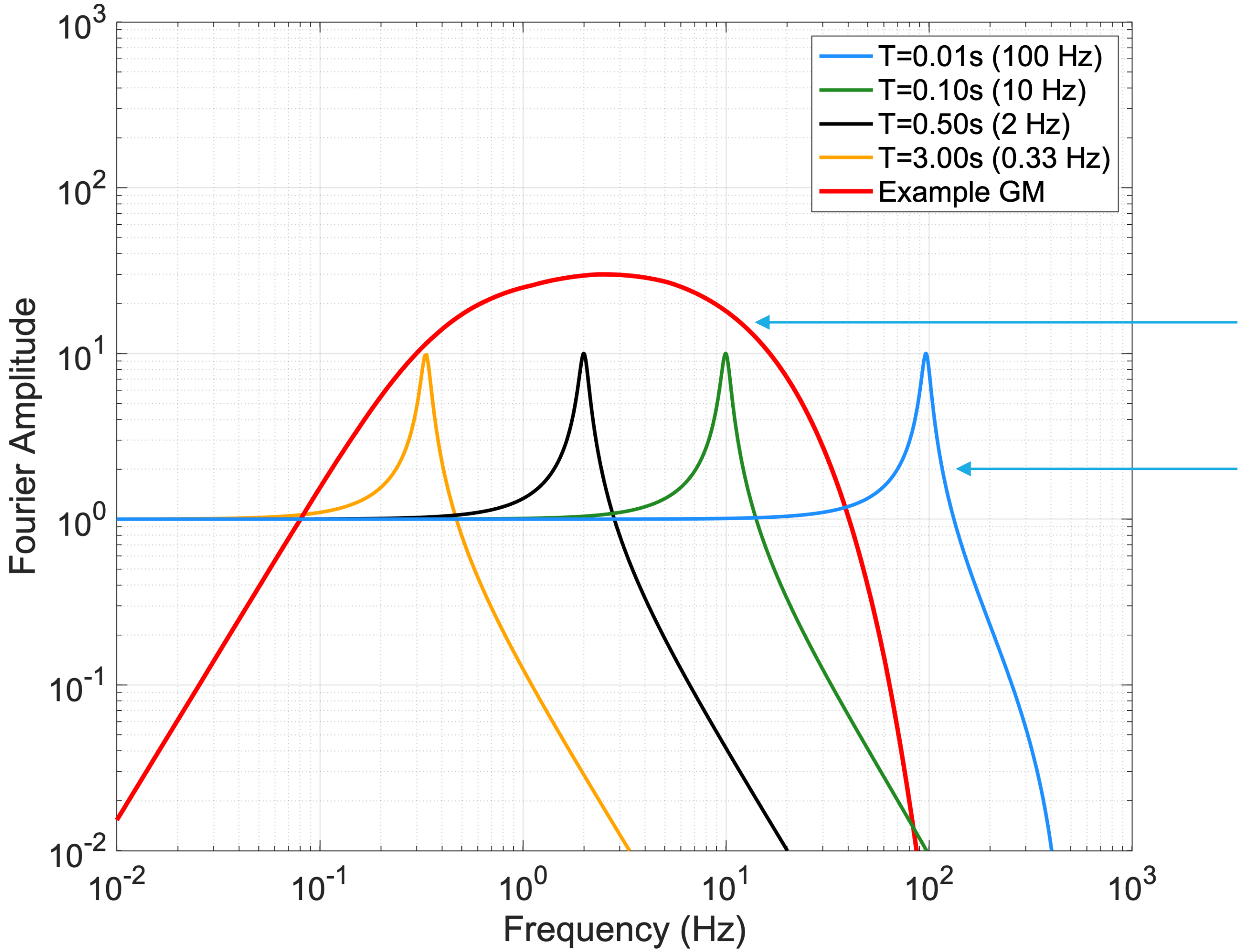
$$|F\{u\}| = |F\{h\}| * |F\{p\}|$$

SDOF transfer function

FAS of the ground motion (GM)

The response spectrum at short periods primarily reflects the strength of the underlying Fourier spectral amplitudes over a broad range of frequencies.

$$|F\{u\}| = |F\{h\}|^{*}\,|F\{p\}|$$

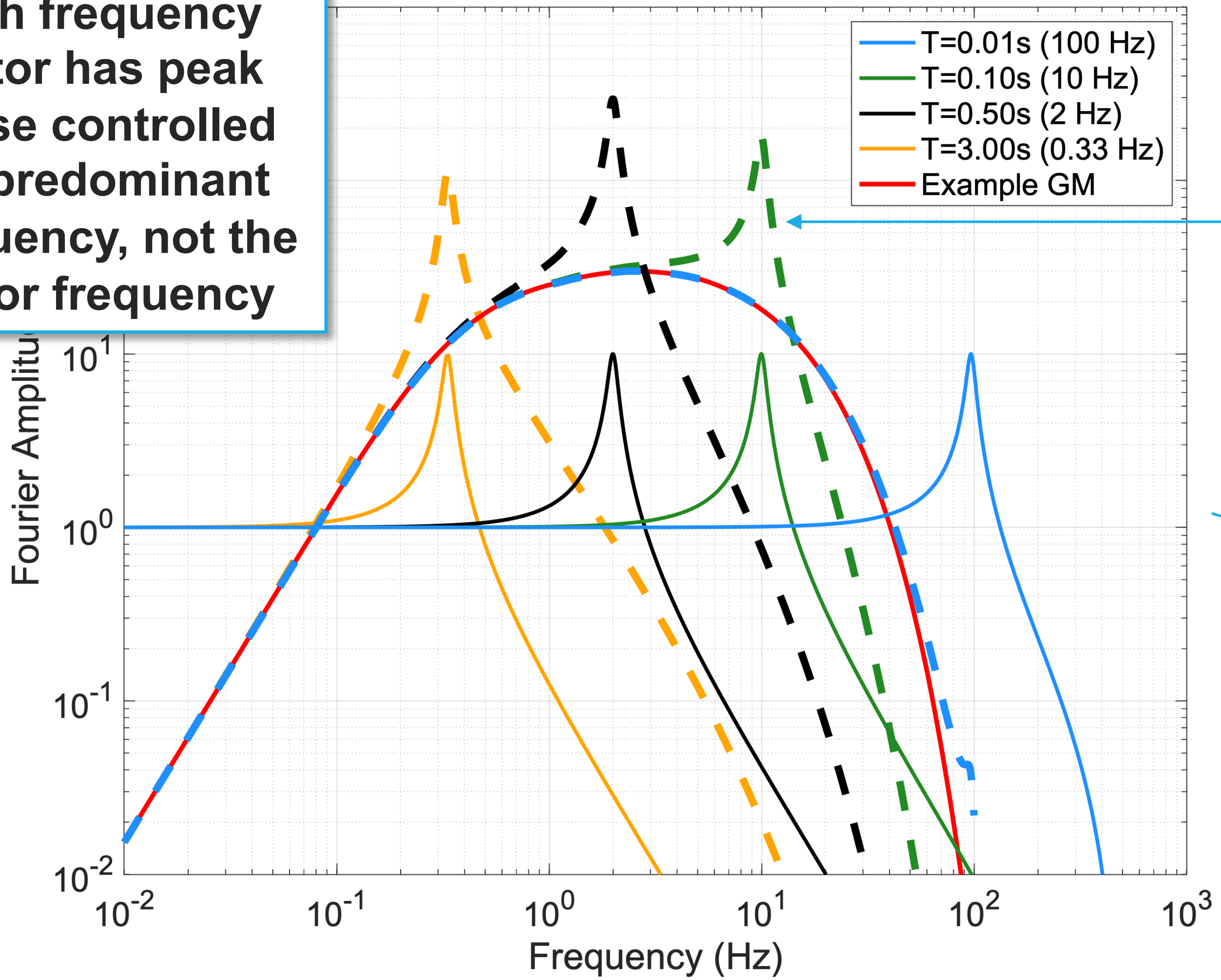


$$|F\{p\}|$$

$$|F\{h\}|$$

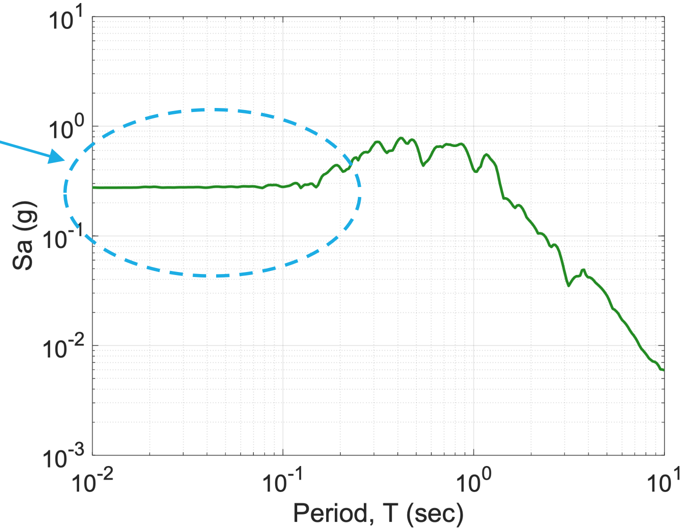
The response spectrum at short periods primarily reflects the strength of the underlying Fourier spectral amplitudes over a broad range of frequencies.

the high frequency oscillator has peak response controlled by the predominant GM frequency, not the oscillator frequency



$$|F\{u\}| = |F\{h\}|^{*}\,|F\{p\}|$$

$$|F\{u\}|$$



Introduction

Advantages of using FAS in ground motion studies:

The scaling of FAS is easier to constrain using seismological theory, and numerous seismological models of the FAS are available.
(e.g., Brune,1970; Boore, 2004)

Linear site response remains linear at all frequencies and does not depend on the spectral content of the input motion, as is the case for response spectra.
(Stafford et al., 2017)

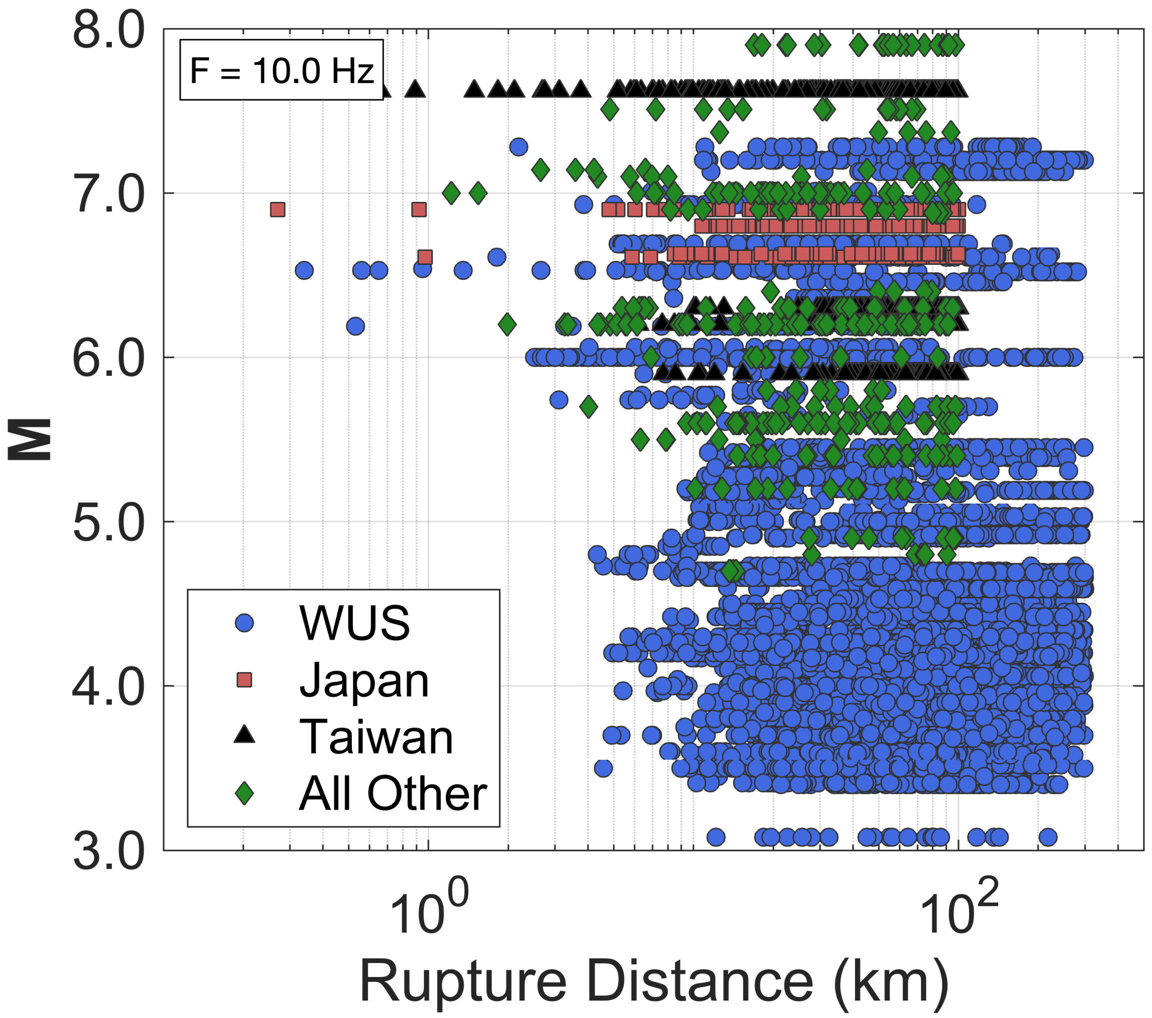
In non-ergodic models, which require ample recorded data, effects from small magnitude events are more easily transferred to larger magnitudes.
(Lavrentiadis et al., 2023)

For calibrating finite-fault simulations, Fourier spectra are more closely related to the physics in the simulations.

Covered in part 2 of this talk.

Ground Motion Model for FAS

Ground Motion Model for FAS



Bayless and Abrahamson (BA18):

empirical ground-motion model (GMM) for shallow crustal earthquakes in California based on the Next Generation Attenuation-West2 database.

For the EAS component of the FAS:

$$EAS(f) = \sqrt{\frac{1}{2}\left[FAS_{HC1}(f)^2 + FAS_{HC2}(f)^2\right]}$$

The EAS is smoothed in log-space.

Ground Motion Model for FAS

$$ln\ EAS = ln\ EAS_{med} + \epsilon\sigma$$

$$ln\ EAS_{med} = f_{Magnitude} + f_{Path} + f_{Site} + f_{Basin} + f_{Depth} + f_{SOF}$$

$$\sigma = \sqrt{\tau^2 + \phi_{S2S}^2 + \phi_{SS}^2}$$

- Non-linear mixed effects regression independently at each frequency
- Performed in a series of regression steps
  - prevents trade-off of correlated model coefficients and to constrain different components of the model using the data relevant to each
- Emphasis is placed on model scaling and extrapolation outside ranges well constrained by data
- Uses finite-fault simulations to constrain near-fault saturation

Ground Motion Model for FAS

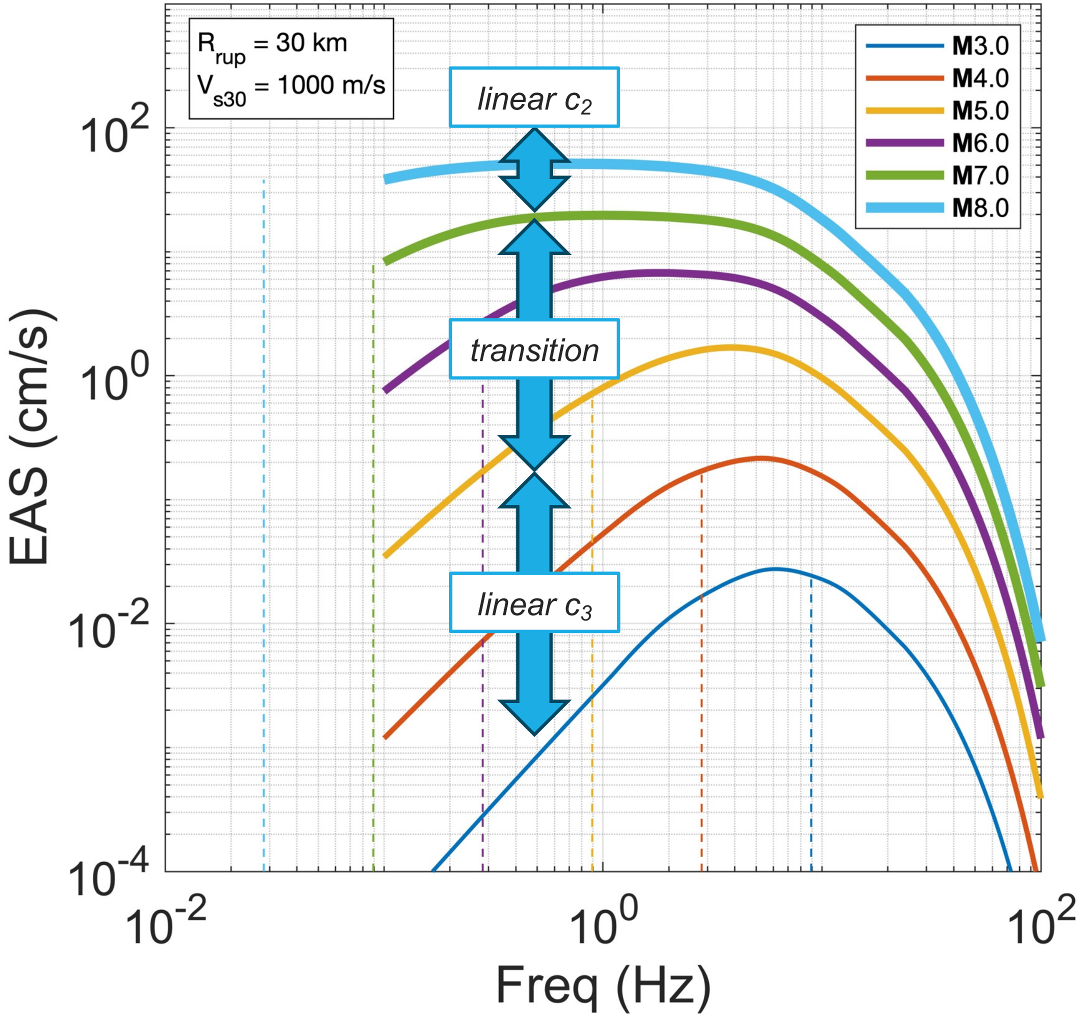
$$f_M = c_1 + c_2(\mathbf{M} - 6) + \frac{(c_2 - c_3)}{c_n}\ln\left(1 + e^{c_n(c_M - \mathbf{M})}\right)$$

c_2 is the frequency-independent linear M-scaling slope for frequencies well above the theoretical corner frequency.

The $(c_2 - c_3)/c_n$ term captures both the linear scaling of the FAS below the theoretical corner frequency (coefficient c_3) and the nonlinear transition to that scaling.

The coefficient c_n controls the width of the magnitude range over which the transition between low- and high-frequency linear scaling occurs;

The coefficient c_M is the magnitude at the midpoint of this transition.



Ground Motion Model for FAS

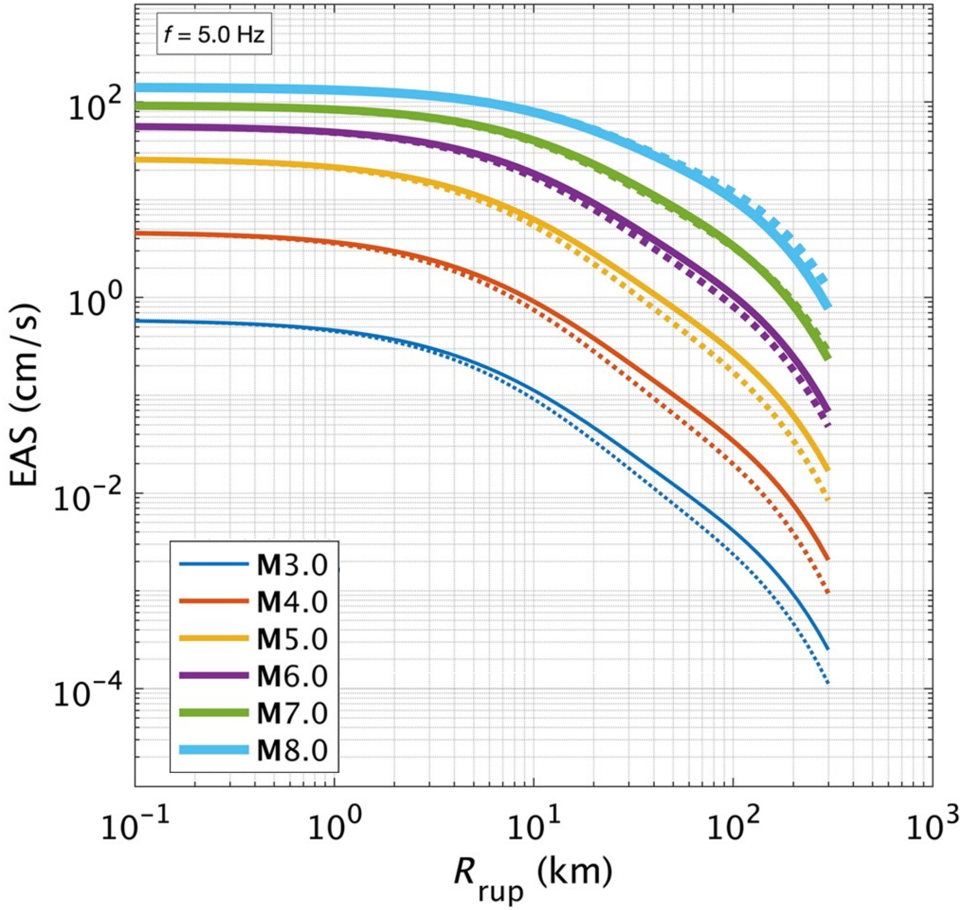
$$f_P = c_4 \ln(R_{\text{rup}} + c_5 \cosh(c_6 \max(\mathbf{M} - c_{hm}, 0))) + (-0.5 - c_4)\ln(\hat{R}) + c_7 R_{\text{rup}}$$

c_4 term: near-source geometric spreading, which is magnitude and frequency dependent.

c_5 term: magnitude and frequency dependence on the geometric spreading

- This additive distance is designed to capture the near-source amplitude saturation effects of the finite-fault rupture dimension.
- largest additive distance at high frequencies.

c_7 term: crustal anelastic attenuation (CA/Nevada)



Ground Motion Model for FAS

$$f_S = f_{\mathrm{SL}} + f_{\mathrm{NL}} + f_{Z1}$$

$$f_{\mathrm{SL}} = c_8 \ln\left(\frac{\min(V_{S30}, 1000)}{1000}\right)$$

$$f_{\mathrm{NL}} = f_2 \ln\left(\frac{I_R + f_3}{f_3}\right),$$

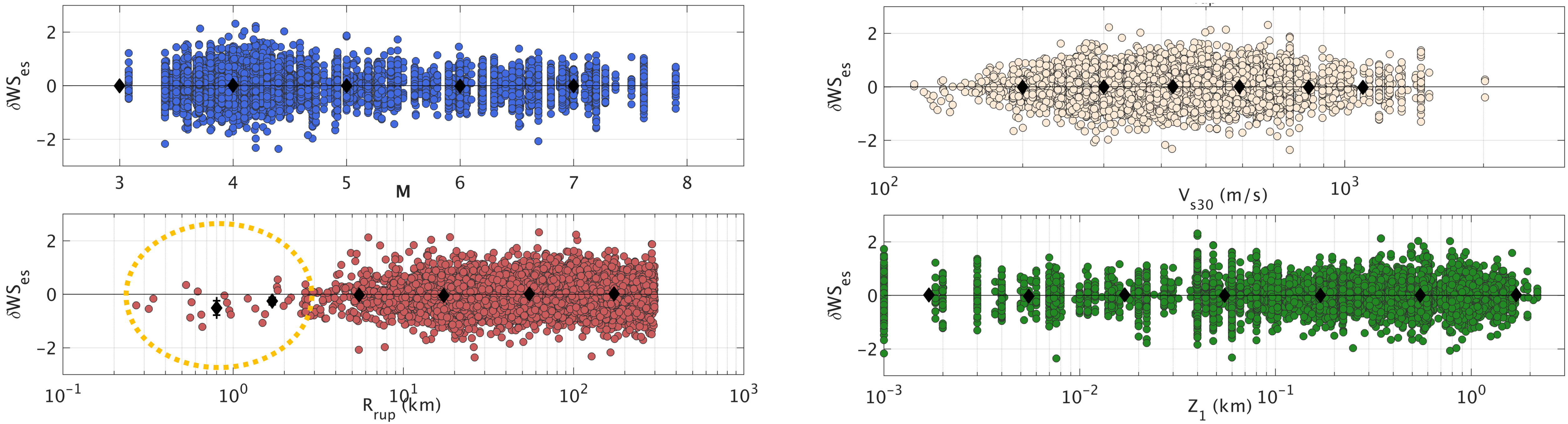
$$f_{Z1} = c_{11} \ln\left(\frac{\min(Z_1, 2.0) + 0.01}{Z_{1\mathrm{Ref}} + 0.01}\right)$$

f_{SL} and f_{Z1} are determined empirically from CA/Nevada

f_{NL} is constrained using a purely analytical model rather than obtaining it from the data (Hashash et al., 2018)

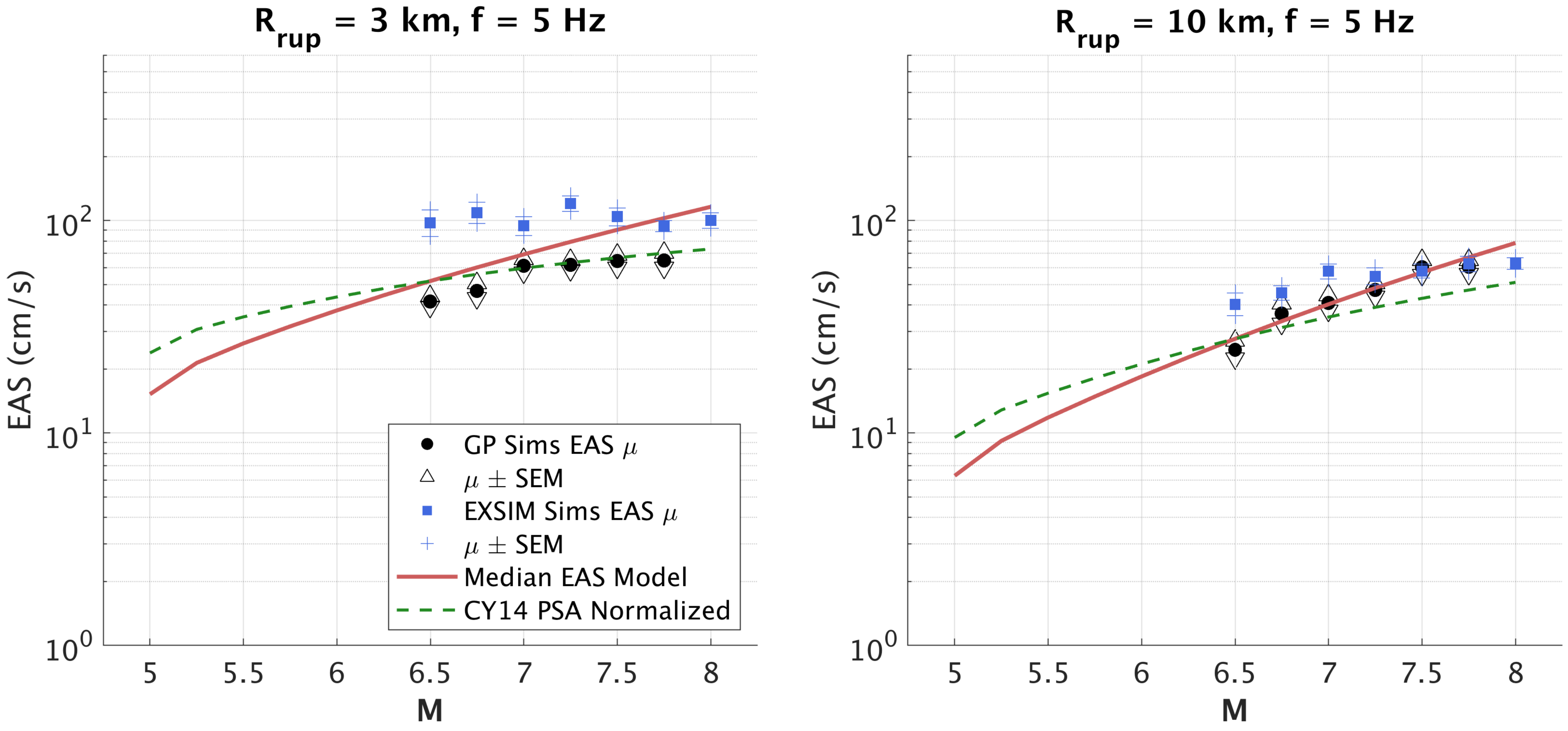
Ground Motion Model for FAS

$$\delta_{\text{total}} = \delta B_e + \delta S2S_s + \delta WS_{es}$$

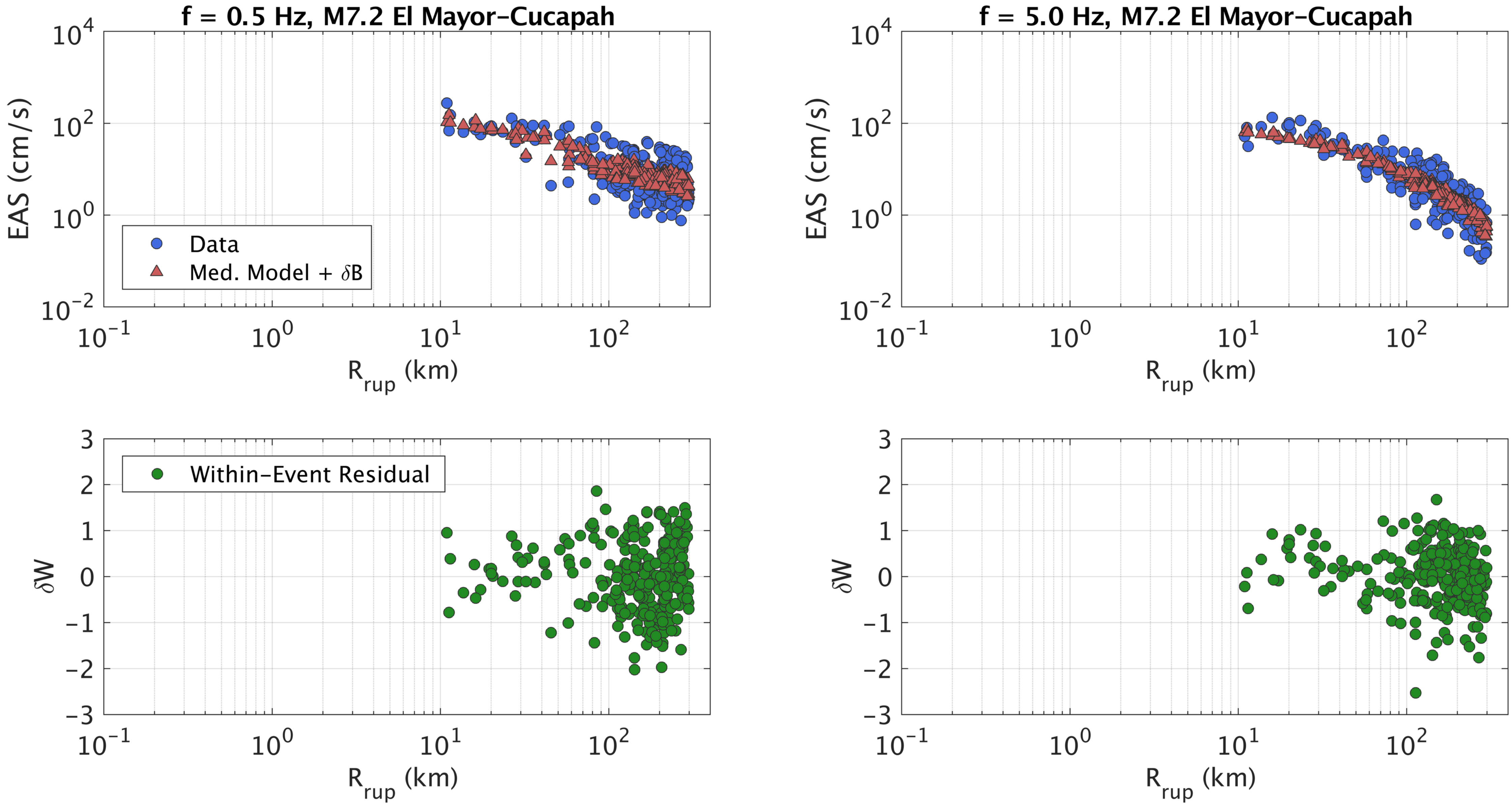


Near-fault saturation in the model is not as strong as implied by the data; intentionally doesn’t allow over-saturation (a peak in distance scaling at some distance greater than zero)

Ground Motion Model for FAS



Ground Motion Model for FAS



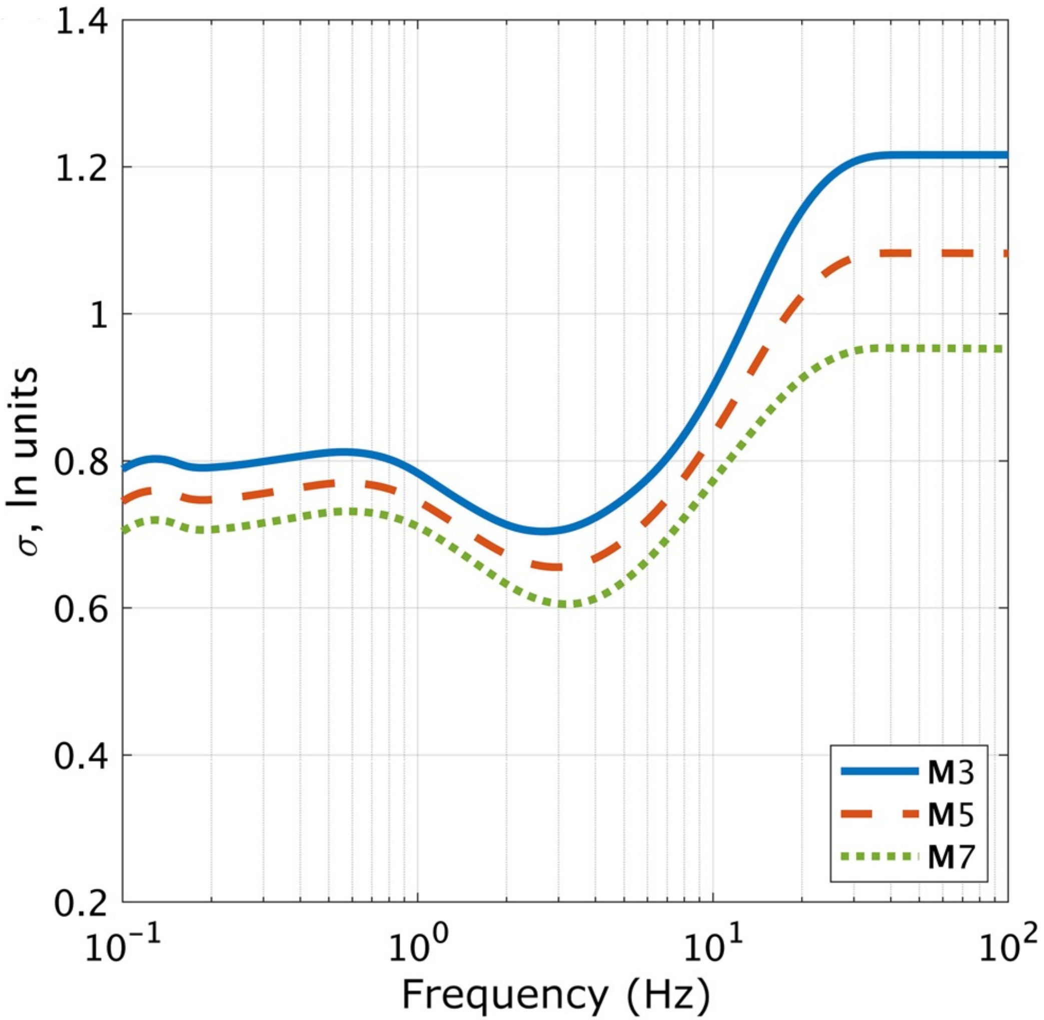
Ground Motion Model for FAS

$$\sigma = \sqrt{\tau^2 + \phi_{S2S}^2 + \phi_{SS}^2}$$

The standard deviation model is frequency and magnitude-dependent:

At low frequencies, the small-magnitude data have higher between-event standard deviation (τ)

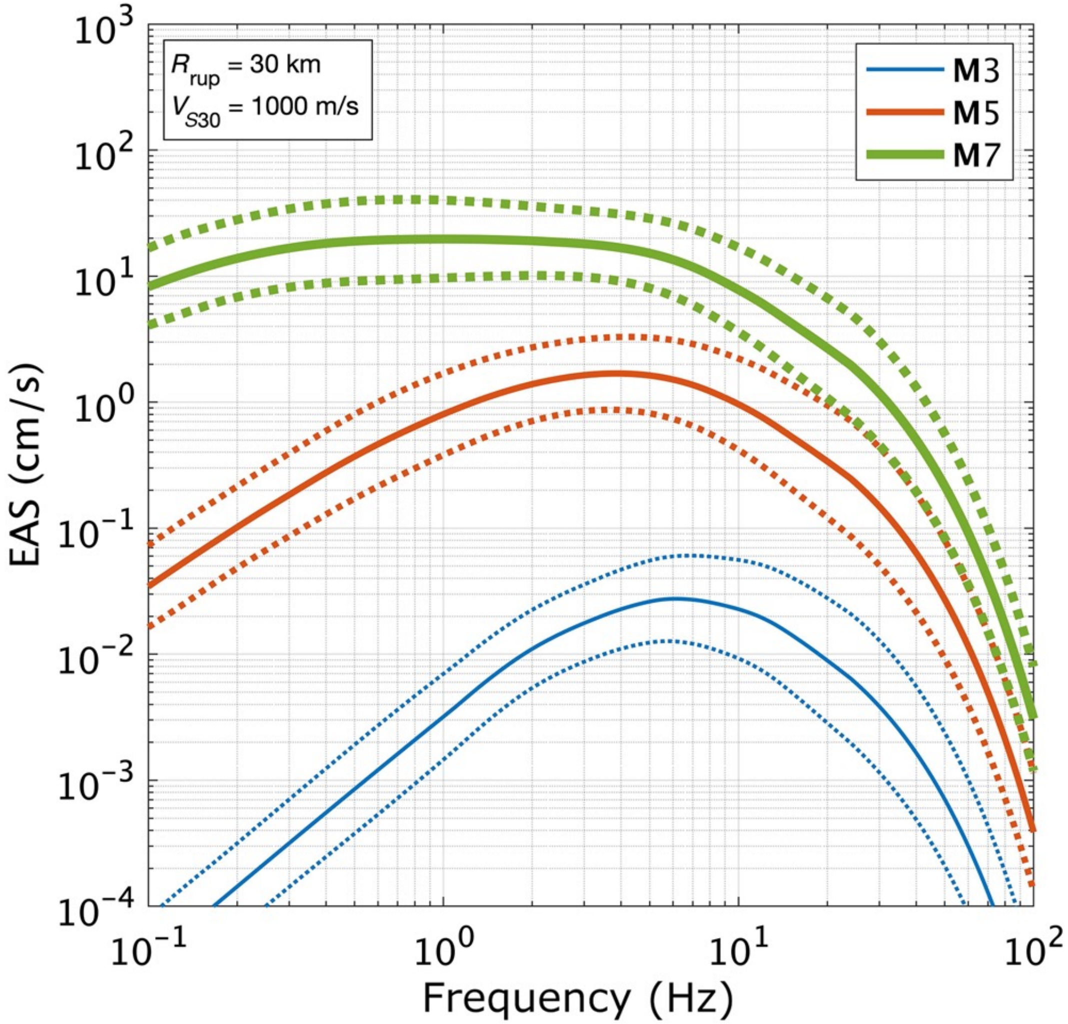
At higher frequencies, not strong magnitude dependence in τ, but ϕ_{SS} and ϕ_{S2S} are larger for the small-magnitude data



Ground Motion Model for FAS

Range of Applicability

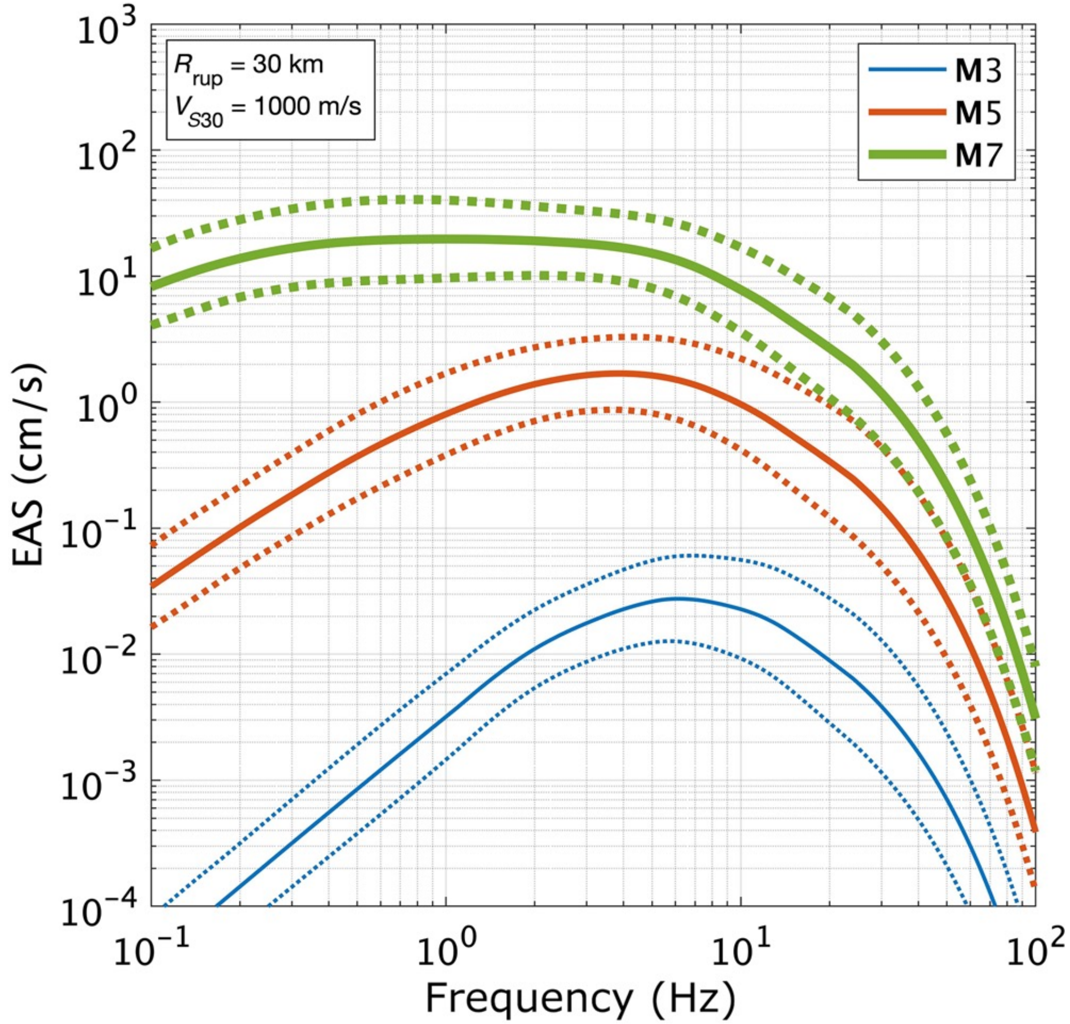
- 0.1 – 100 Hz; kappa-based extrapolation beyond 24 Hz
- Regionalized for CA/Nevada, uses global data to constrain the magnitude scaling and geometric spreading
- Rupture Distances 0 – 300 km
- **M** 3-8
- Vs30 from 180 – 1500 m/s



Ground Motion Model for FAS

Future Updates

- Hanging wall model
- Rupture directivity
- Aftershocks
- Improved site response models
- Wider frequency range
- Partially non-ergodic for other regions
- Non-ergodic (Lavrentiadis et al., 2023)



Inter-frequency correlation of the FAS

Inter-frequency Correlation

Epsilon (ϵ) is the number of standard deviations difference between the observed GM and the median model prediction (ln units)

observed

model median

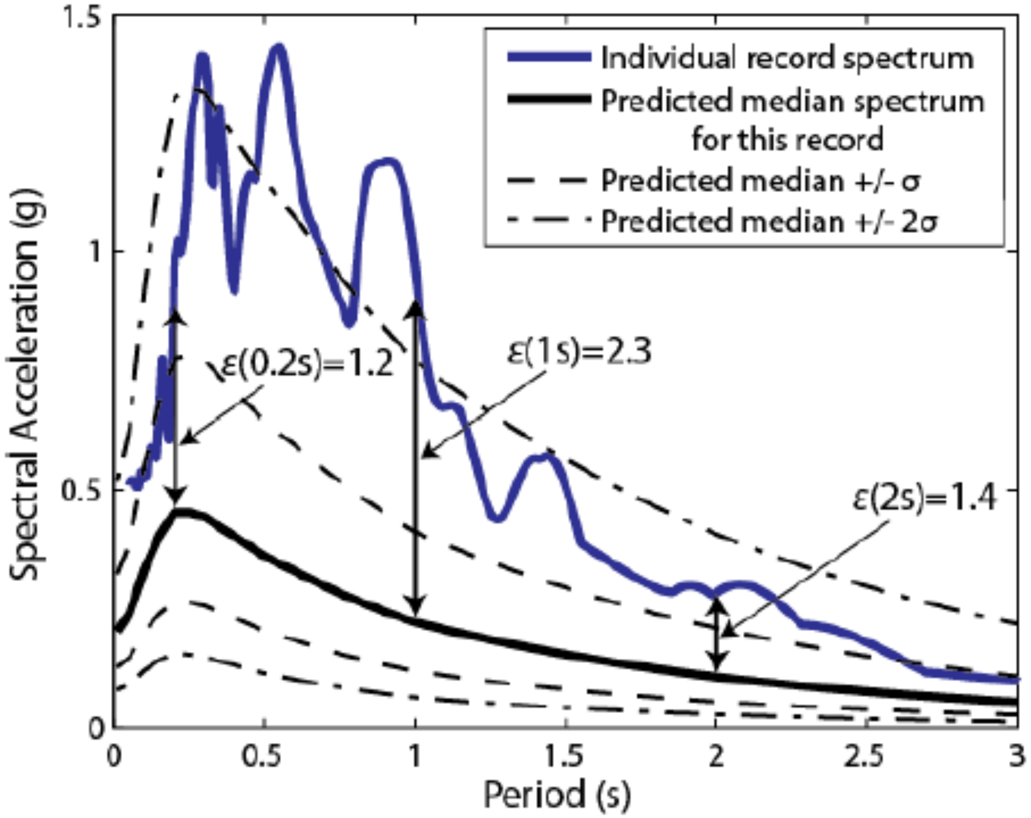
$$\varepsilon(T) = \frac{\ln Sa(T) - \mu_{\ln Sa}(M,R,T)}{\sigma_{\ln Sa(T)}}$$

model sigma

ϵ is a normalized residual

ϵ is **correlated** between spectral periods

The correlation coefficient (ρ_ϵ) is a measure of the linear relationship of random variables ϵ at two frequencies

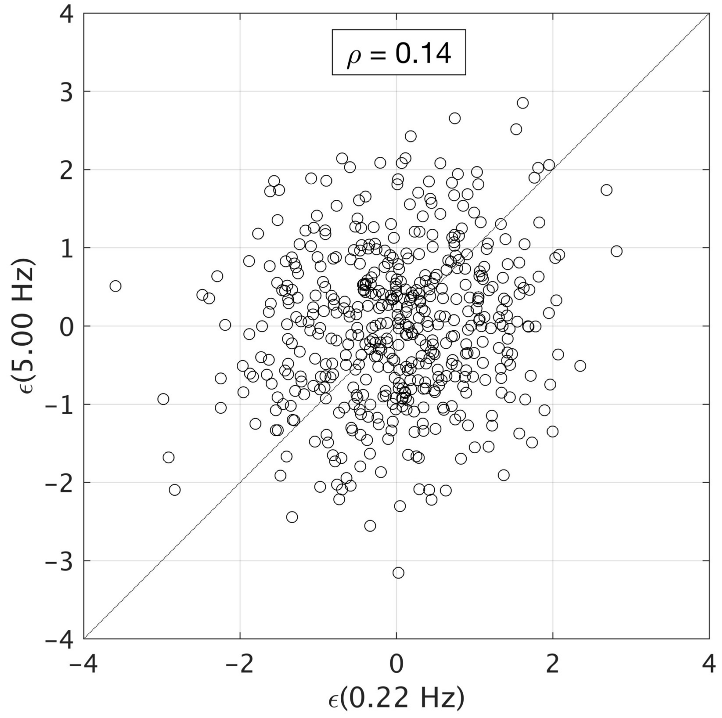


(Baker 2010)

Inter-frequency Correlation

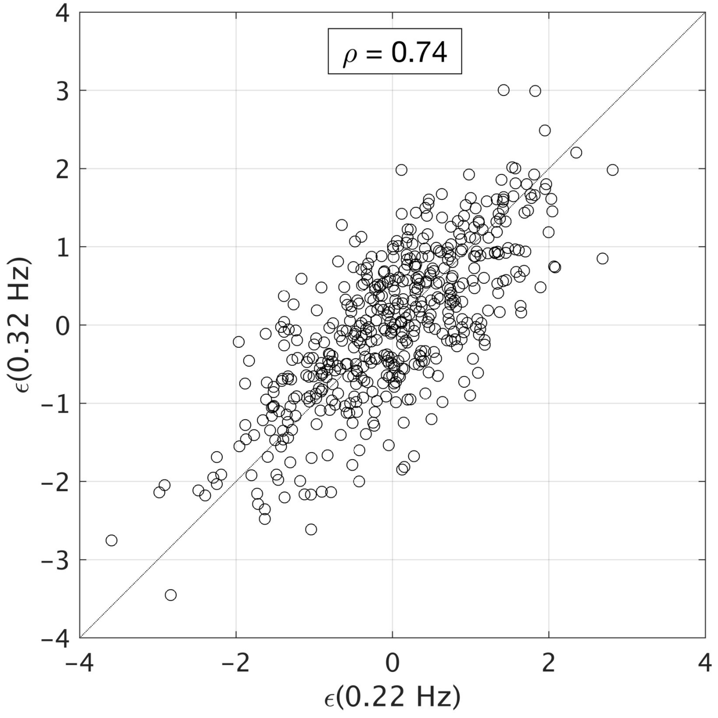
$F_1 = 0.2$ Hz, $F_2 = 5$ Hz

$\rho = 0.14$



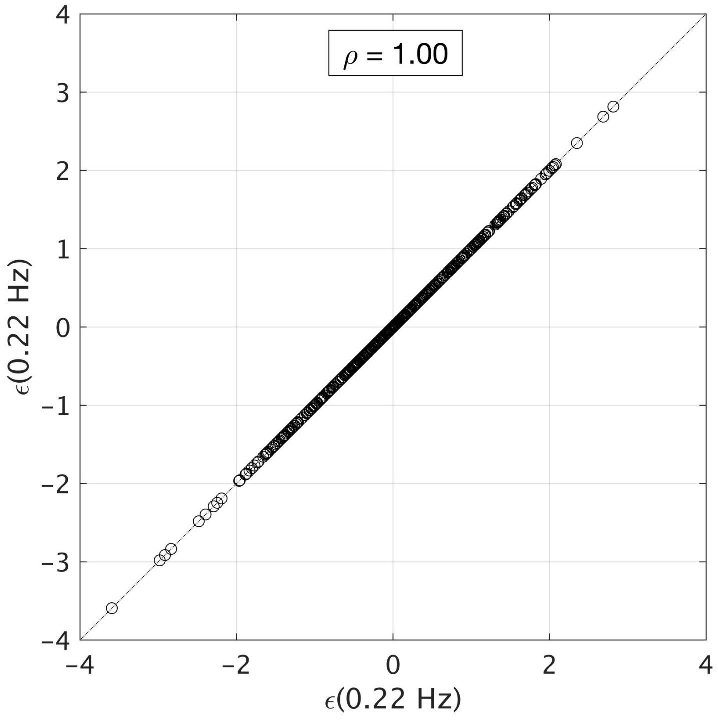
$F_1 = 0.2$ Hz, $F_2 = 0.3$ Hz

$\rho = 0.74$



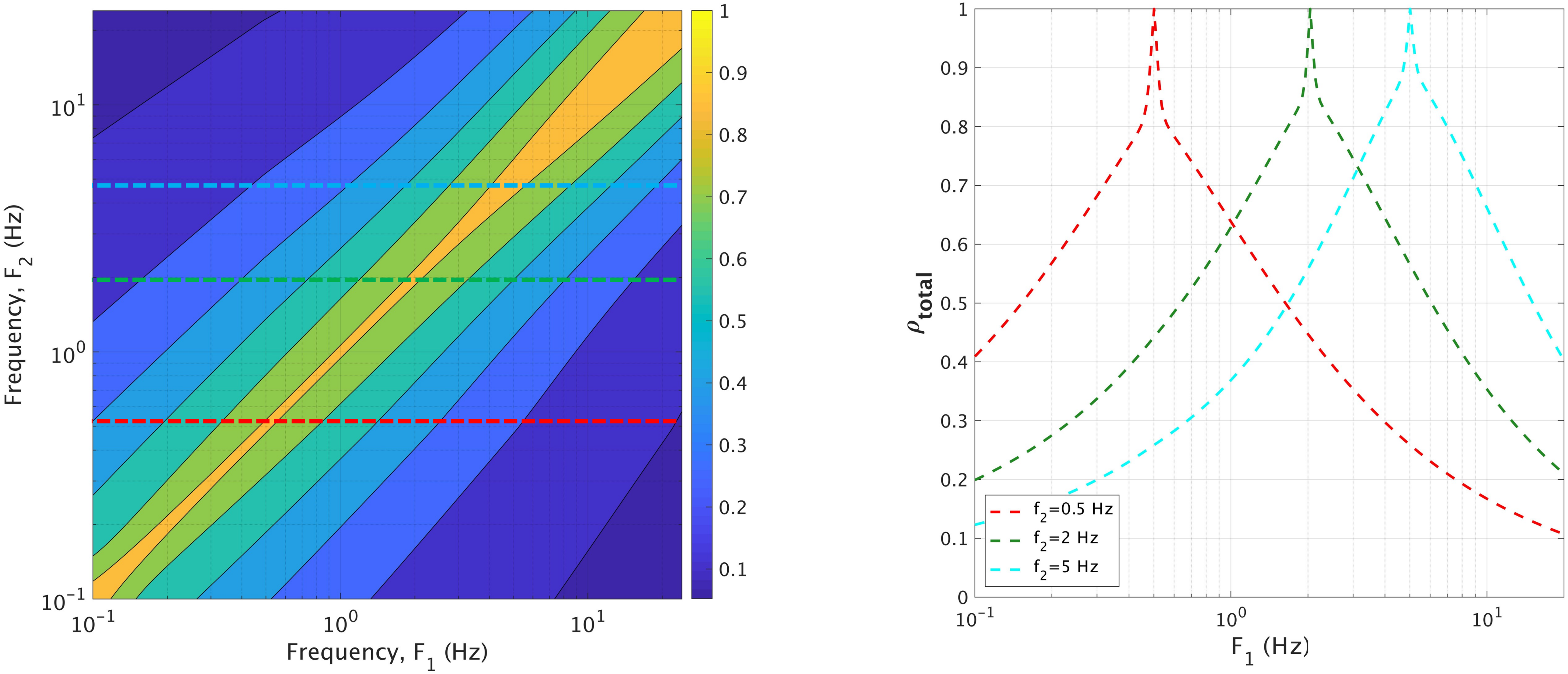
$F_1 = 0.2$ Hz, $F_2 = 0.2$ Hz

$\rho = 1.0$

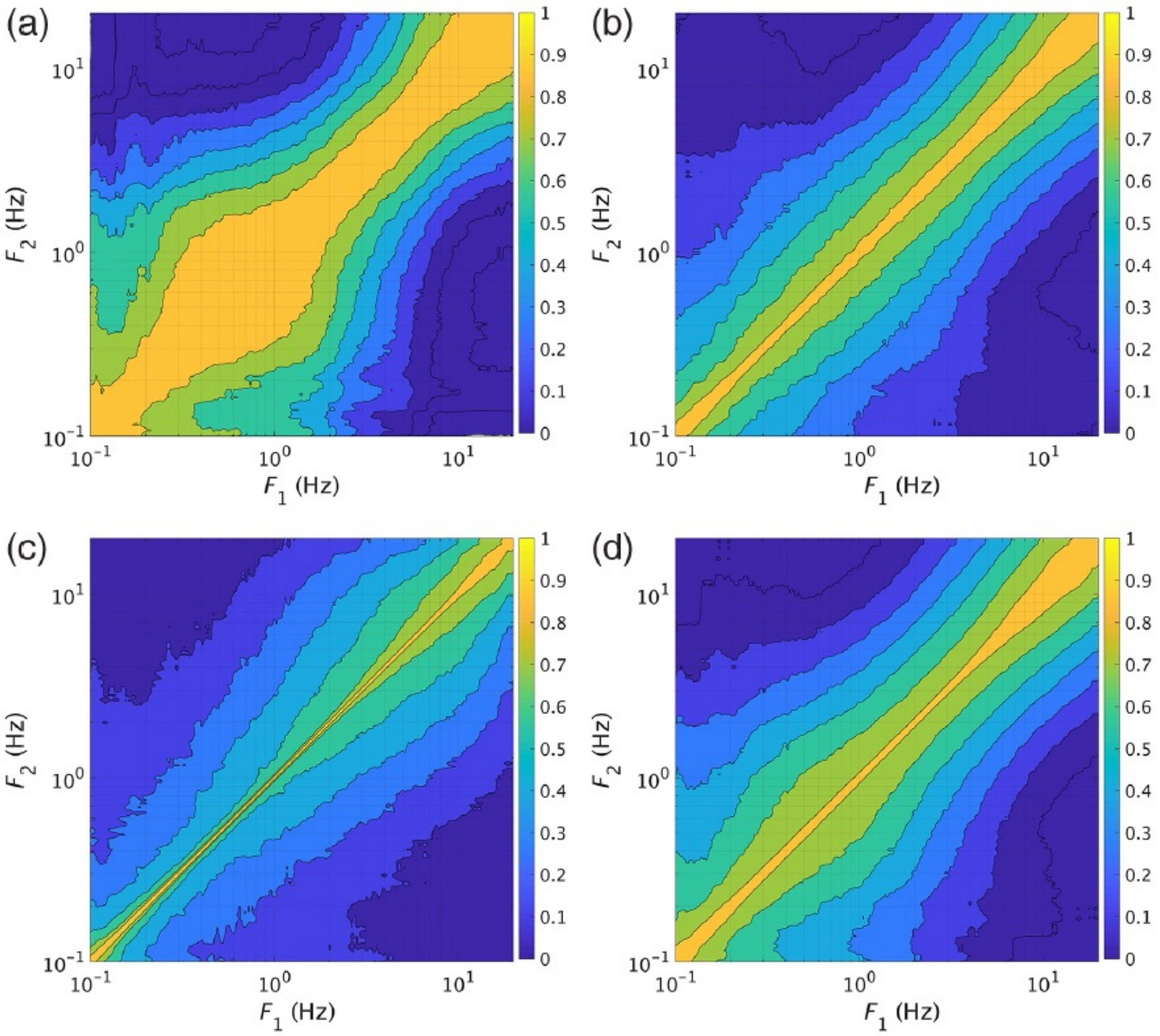


Inter-frequency Correlation

Repeat this calculation of ρ for each period (or frequency) pair of interest.



Inter-frequency Correlation



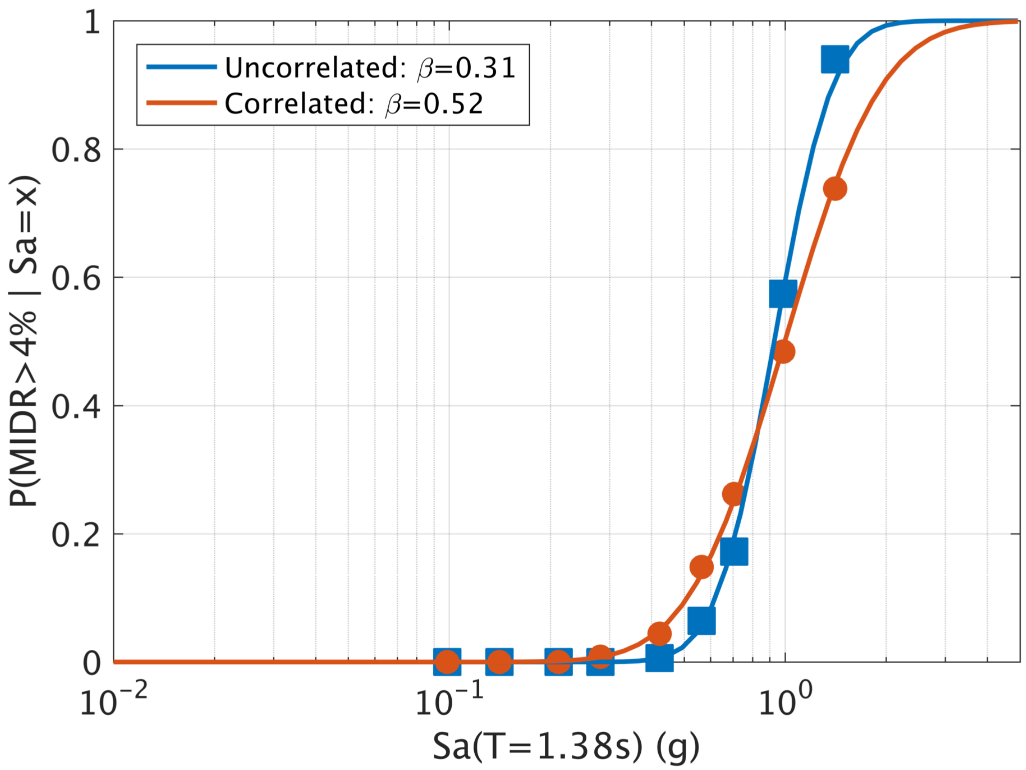
Contours of ρ_ϵ directly from Bayless and Abrahamson (2018) EAS residuals:

- (a) between-event, δB_e
- (b) between-site, $\delta S2S_s$
- (c) within-site, δWS_{es}
- (d) total ρ_ϵ

Significance of the Inter-frequency Correlation

As a measure of the width of spectral peaks, ρ_ϵ is relevant in dynamic structural response.

- Structures are sensitive to a range of frequencies about the fundamental one, especially for nonlinear response.
- ρ_ϵ is related to the width of spectral peaks and troughs (higher correlation – wider)
- Breadth of spectral peaks influences variability of the response.
- Increased ρ_ϵ leads to larger response variability, flatter fragility curves (blue), and in turn, higher risk
- Therefore, ρ_ϵ influences seismic risk and is an important metric for **calibration and validation** of simulations (Bayless and Abrahamson, 2018)

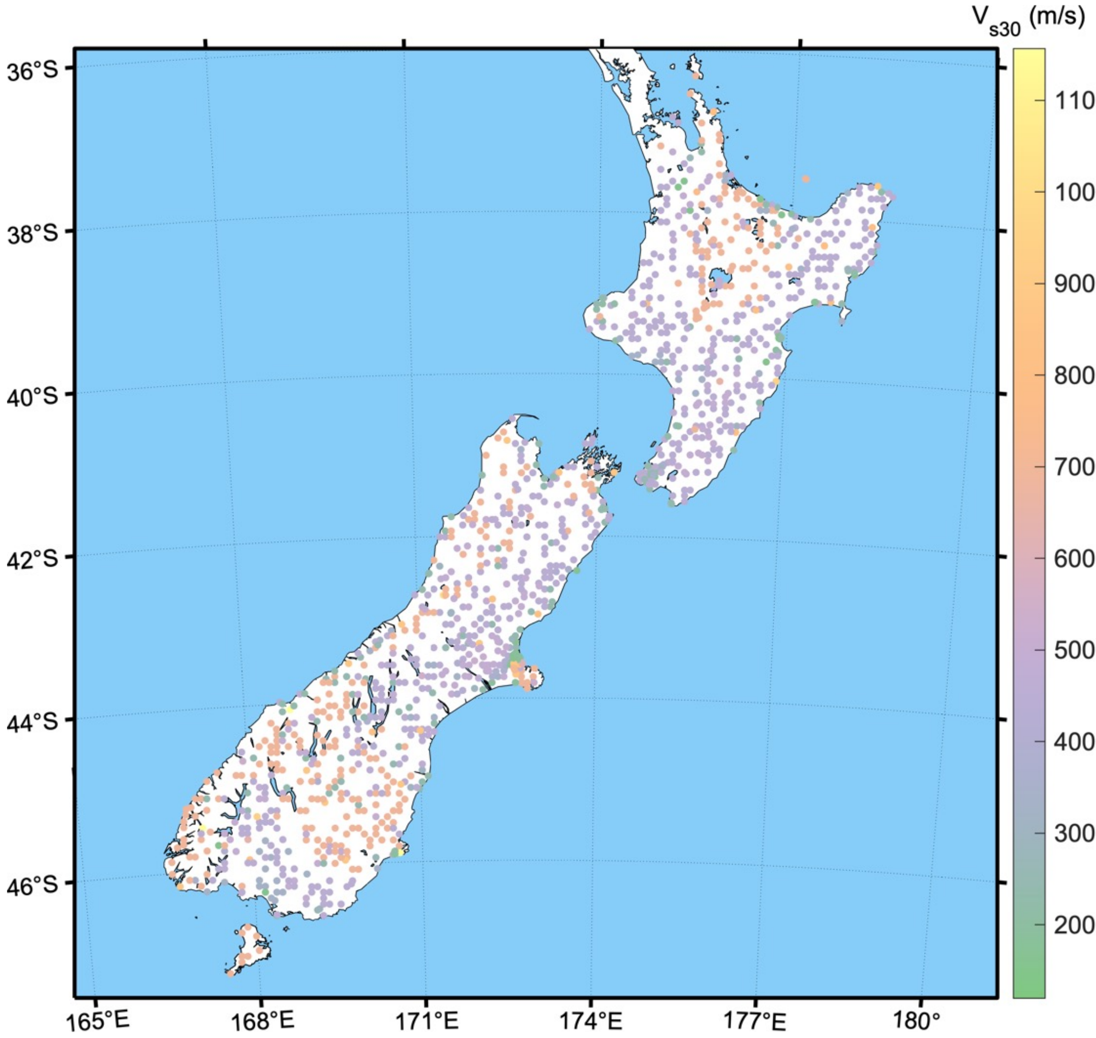


Inter-frequency Correlation in Simulations

CyberShake NZ: crustal earthquake simulations (Bradley et al., 2020)

Evaluation Procedure

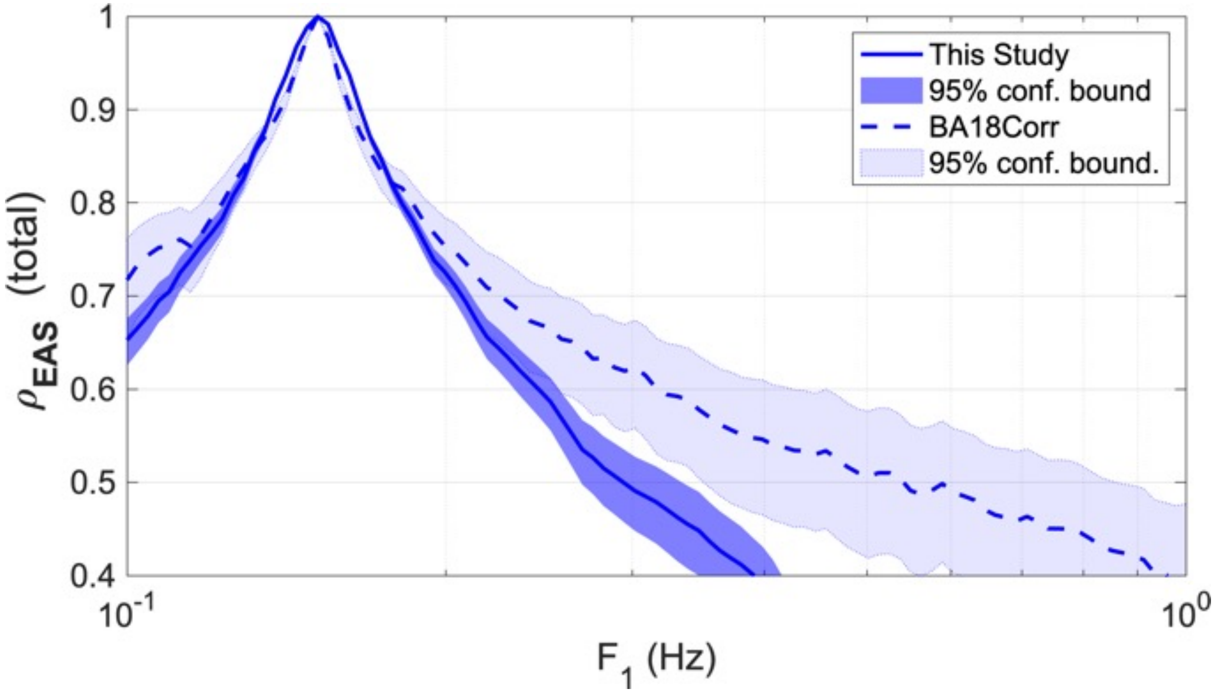
- Calculate residuals from the simulations relative to the FAS model
- Partition the residuals into components: mean bias, between-event, between site, within-site
- Calculate ρ_ϵ for each FAS residual component and the total ρ_ϵ
- Compare with ρ_ϵ from BA18

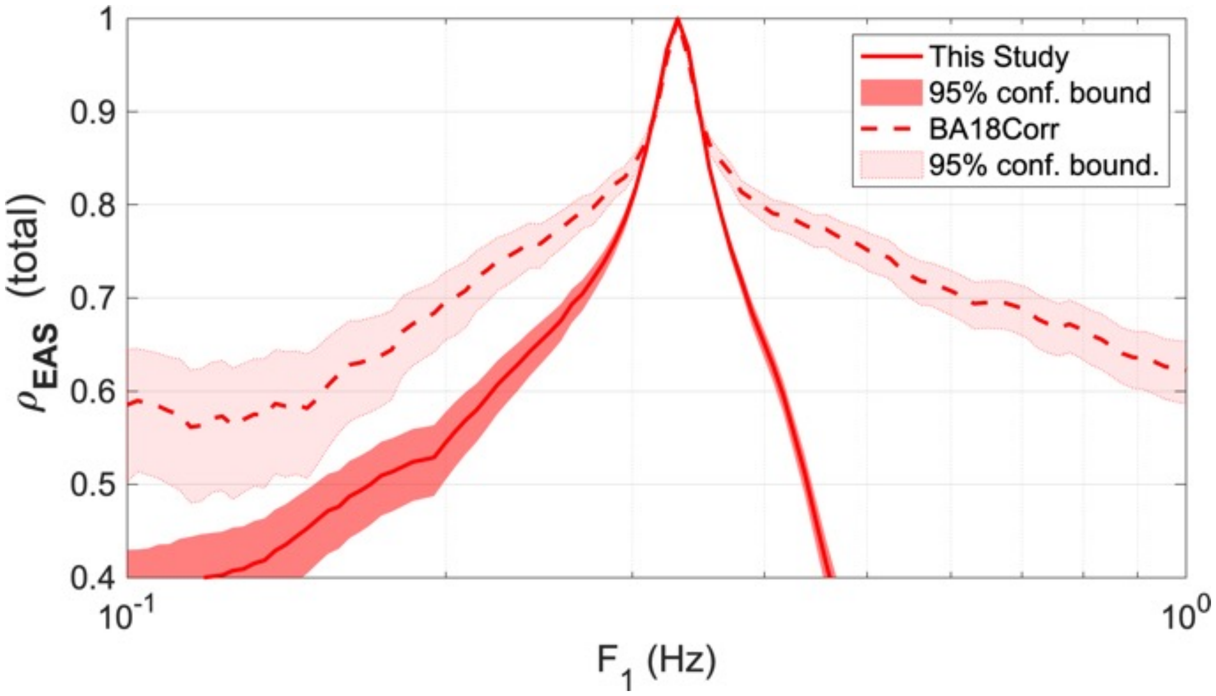


Inter-frequency Correlation: CyberShake NZ

f = 0.15 Hz

f = 0.33 Hz





Inter-frequency Correlation: CyberShake NZ

Summary of Performance

The between-site residuals represents the systematic deviation of the observed amplification at a site from the median amplification predicted by the model.

- Generally, are too high in the simulations.
- Impacted by small variability in CyberShake Vs profiles compared with the profiles for the recorded data.
- **Calibration will involve testing simulations with refined velocity models.**

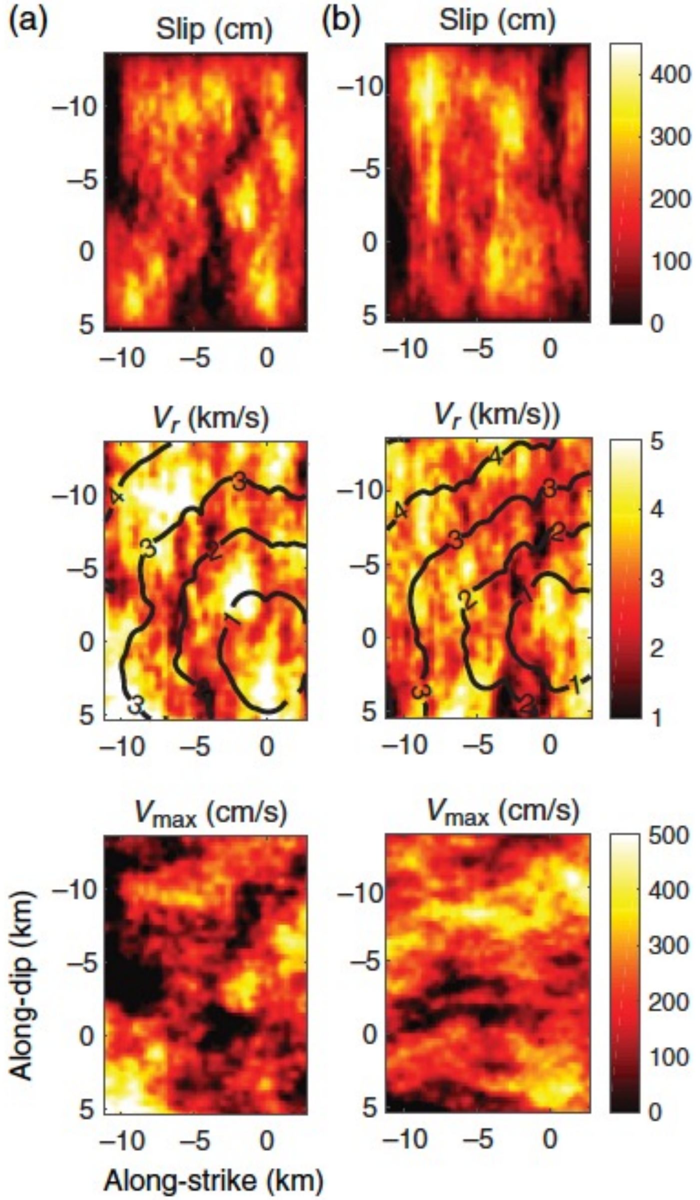
The between-event residuals physically relate to source effects (e.g. stress drop) which drive ground motions over a broad frequency range, and have relatively broad ρ_{ϵ}

- Below about 0.5 Hz performance is good. Above 0.5 Hz the correlation is low.
- **Calibration involves adjustments to source models**

Inter-frequency Correlation: Calibration

Song et al. (2020)

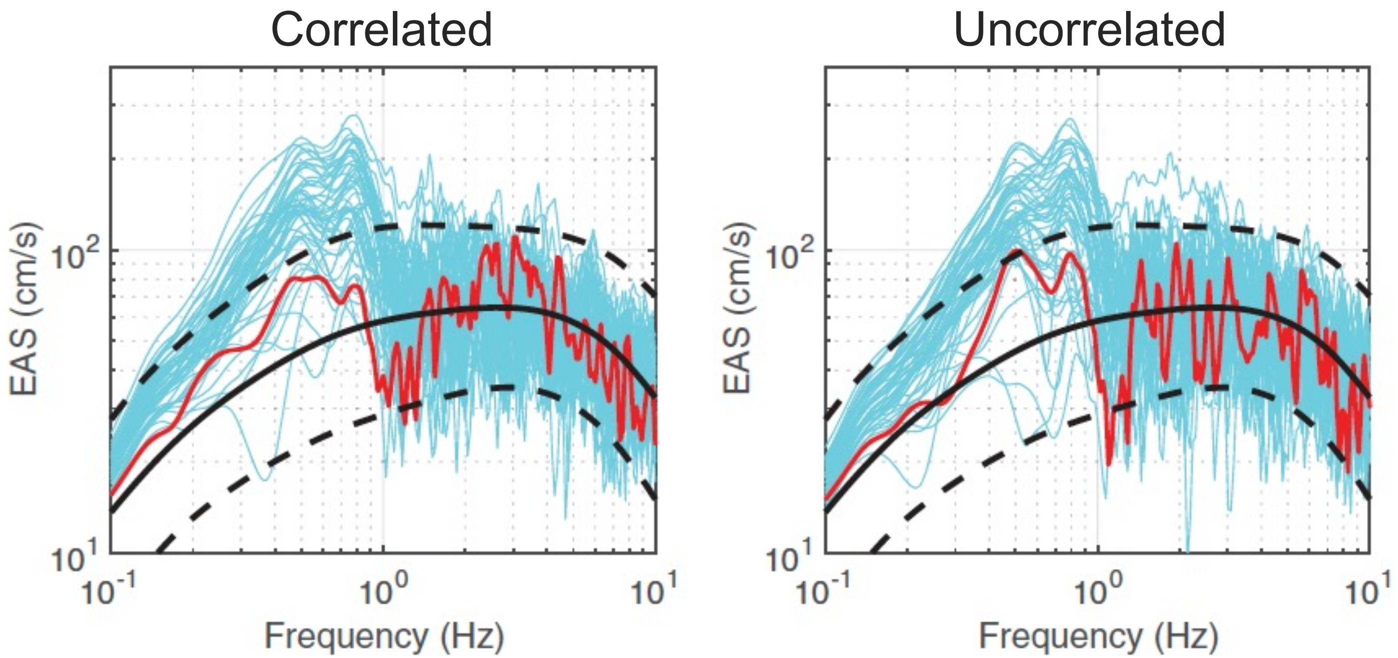
- investigated the effect of pseudo-dynamic source models on the inter-frequency correlation of ground motions with the 1994 Northridge, California, earthquake.
- Pseudo-dynamic emulates the essential physics of earthquake rupture dynamics by statistically analyzing many dynamic rupture models
- The model in (b) have cross correlation between earthquake source parameters (slip, rupture velocity, and peak slip velocity)

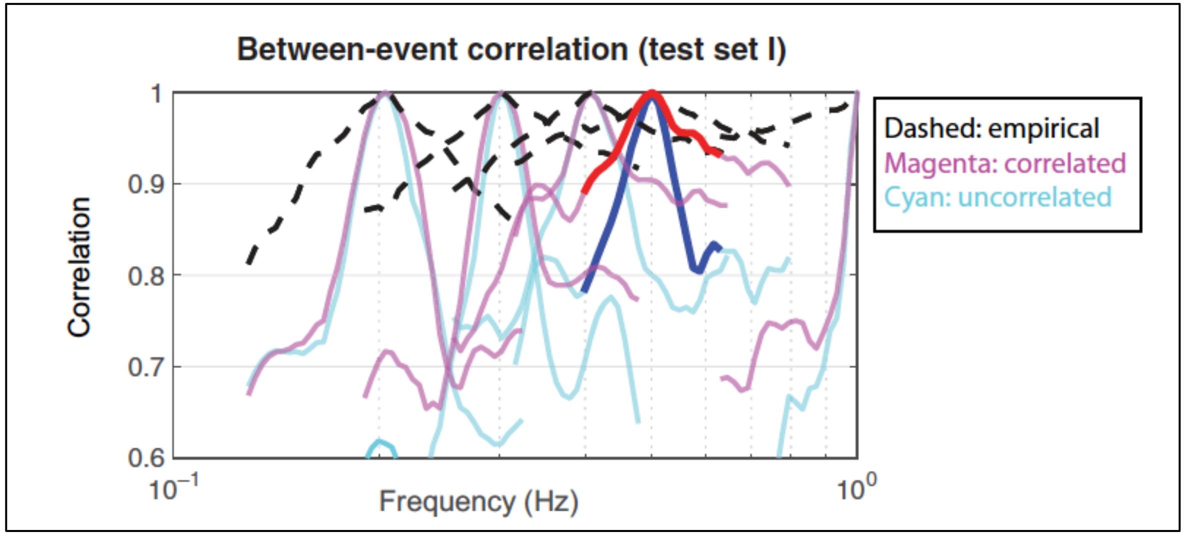


Inter-frequency Correlation: Calibration

Song et al. (2020)

- SCEC Broadband Platform Simulations
- The affected on ρ_ϵ is in the frequency around 0.5 Hz, with little impact on other frequency ranges
- Little impact on the standard deviation
- More work is needed to understand the relation between physics-based earthquake source models and the inter-frequency correlation of ground motions





Summary

The response spectrum at short periods primarily reflects the strength of the underlying Fourier spectral amplitudes over a broad range of frequencies.

As a result, there are numerous advantages to using FAS in ground-motion studies.

One advantage is for calibration and validation of earthquake ground motion simulations, which can be used in future seismic hazard and risk assessments. More research on this topic is needed.

Thank you

jeff.bayless@aecom.com