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An Empirical Model for the Inter-Frequency Correlation of Epsilon for Fourier Amplitude Spectra

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Abstract

An empirical ground motion model (GMM) is presented for the inter-frequency correlation of normalized residuals, epsilon (ϵ), for smoothed Fourier amplitude spectra (FAS). The inter-frequency correlation of ϵ (ρ_ϵ) model is developed for the smoothed effective amplitude spectrum (EAS), as defined by PEER (Goulet et al., 2018). The EAS is the orientation-independent horizontal component FAS of ground acceleration. Ground-motion data are from the Pacific Earthquake Engineering Research Center (PEER) Next Generation Attenuation-West 2 (NGA-West2) database (Ancheta et al., 2014), which includes shallow crustal earthquakes in active tectonic regions.

To develop the model, normalized residuals are obtained from the Bayless and Abrahamson (2018) GMM for EAS. The residuals are partitioned into between-event, between-site, and within-site components, and a model is developed for the total correlation between frequencies. The total correlation model features a two-term exponential decay with the natural logarithm of frequency. At higher frequencies, the model differs substantially from previously published models, where the smoothing of the EAS has a large effect on the resulting correlations.

The empirical ρ_ϵ are not found to have statistically significant magnitude, distance, site parameter, or regional dependence, although potential regional variations should be studied further. The model is applicable for crustal earthquakes in active tectonic regions worldwide, for rupture distances of 0 – 300 km, $\boldsymbol{M}$ 3.0 – 8.0, and for frequencies 0.1 – 24 Hz.

(1) Background

Using the Al Atik et al., (2010) notation, ground-motion residuals may be partitioned into δB, the between-event residual, $\delta S2S$, the site-to-site residual, and δWS, the single-station within-event residual. The residual components are converted to epsilon (ϵ_B, ϵ_{S2S}, and ϵ_{WS}) by normalizing the residuals by their standard deviations (τ, ϕ_{S2S} and ϕ_{ss}, respectively).

For a given recording, the values of ϵ at neighboring periods (T) are generally correlated. For example, if a ground motion is stronger than average at T=1.0 s, then it is likely to also be stronger than expected at nearby periods, e.g. T=0.8 s or T=1.2 s; however, for a widely-spaced period pair, the ϵ values will be weakly correlated. The inter-period (or equivalently, inter-frequency) correlation coefficient, ρ, quantifies the relationship of ϵ values between periods for a given recording.

To account for all residual terms, the total correlation is calculated as:

$$\rho_{total}(f_1,f_2) = \frac{\rho_B(f_1,f_2)\tau(f_1)\tau(f_2) + \rho_{S2S}(f_1,f_2)\phi_{S2S}(f_1)\phi_{S2S}(f_2) + \rho_{WS}(f_1,f_2)\phi_{ss}(f_1)\phi_{ss}(f_2)}{\sigma(f_1)\sigma(f_2)}$$

Motivation

The parameter ϵ is an indicator of the peaks and troughs at a given frequency in a spectrum, and ρ_ϵ characterizes the relative width of these extrema. The width of peaks and troughs have significance in risk assessments involving simulated ground motions, because the variability in the dynamic structural response can be under-estimated if ρ_ϵ in simulated ground motions is too low (Bayless and Abrahamson, 2018).

The model is created for Fourier amplitudes because the FAS is a more direct representation of the frequency content of the ground motions than PSA and is generally better understood by seismologists. This leads to several advantages, both in the empirical modeling and in forward application of ρ_ϵ.

(2) Model Description

Ground-Motion Data

- **Database**: subset of the NGA-West2 database, as described in Bayless and Abrahamson (2018).
- **Intensity Measure**: effective amplitude spectrum (EAS), as defined by PEER (Goulet et al., 2018) with smoothing using the $\log_{10}$-scale Konno and Ohmachi (1998) smoothing window ; consistent with the PEER database and with other PEER projects.
- **Residuals**: from the Bayless and Abrahamson (2018) GMM for EAS.

Formulation

- The total ρ_ϵ and ρ_ϵ for each residual component are shown in Figure 1 (contours of the coefficient) and Figure 2 (cross-sections of the contours at frequencies 0.2, 0.5, 2, 5, and 15 Hz).
- The model for the total ρ_ϵ is shown in Figures 3 and 4. The model is a is a two-term exponential decay with the natural logarithm of frequency, given by:

$$\rho_{\epsilon,total,Model}(f_1,f_2) = tanh[A(f_m)e^{B(f_m)*f_r} + C(f_m)e^{D(f_m)*f_r}]$$
$$f_r = \left|ln\left(\frac{f_1}{f_2}\right)\right| ; \quad f_m = min(f_1,f_2)$$

where f_1 and f_1 are the two frequencies considered, $tanh$ is the hyperbolic tangent, A, B, C, and D are frequency-dependent constants, Two exponential terms are required to model the shape of the correlation cross-sections (Figure 4) which starts off with a steep decay at frequencies very close to the conditioning frequency, and then flattens as the log ratio of frequencies increases.

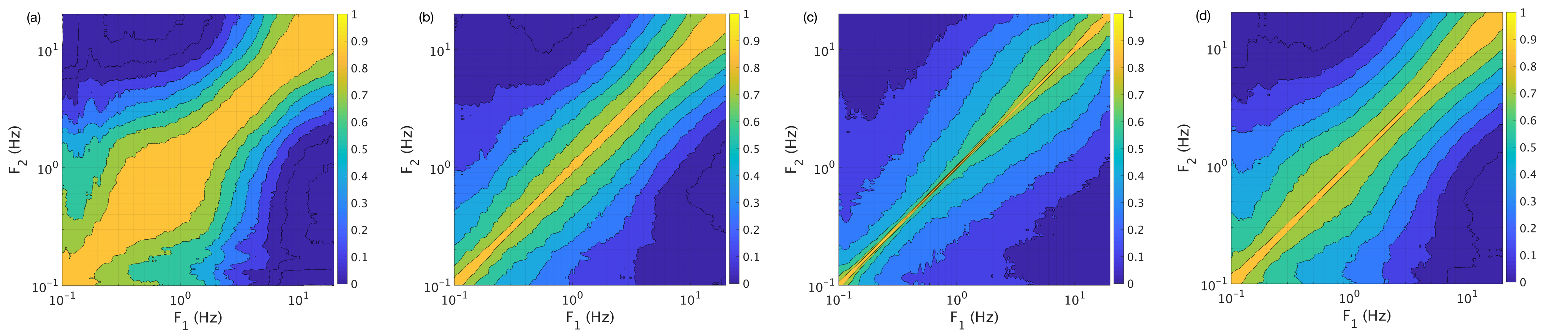

Figure 1: Empirical ρ_ϵ contours, showing (a) the between-event component, (b) the between-site component, (c) the within-site component, and (d) the total

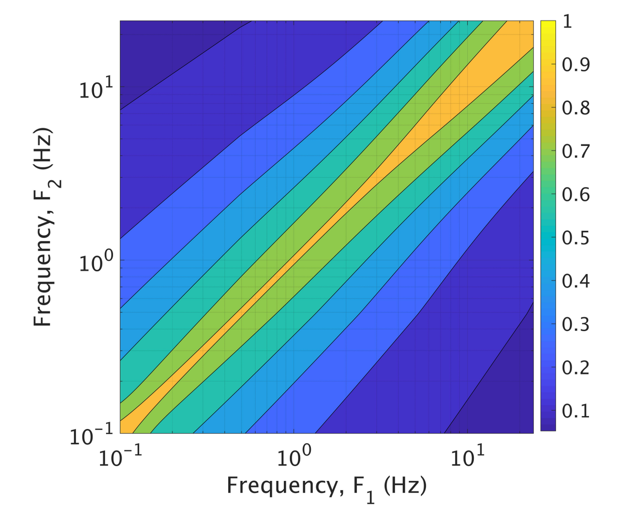

Figure 3: Model total ρ_ϵ contours

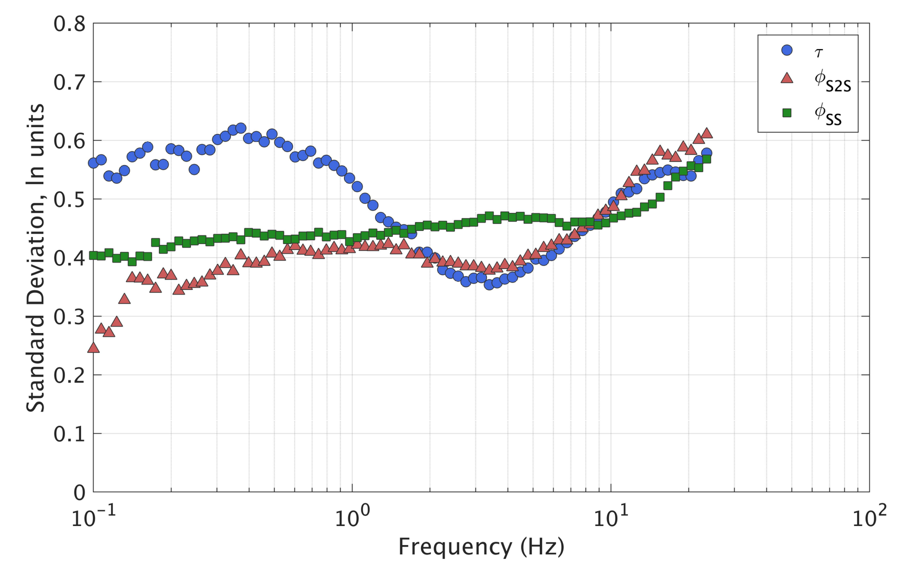

Figure 5: Standard deviation components of the Bayless and Abrahamson (2018) EAS GMM residuals

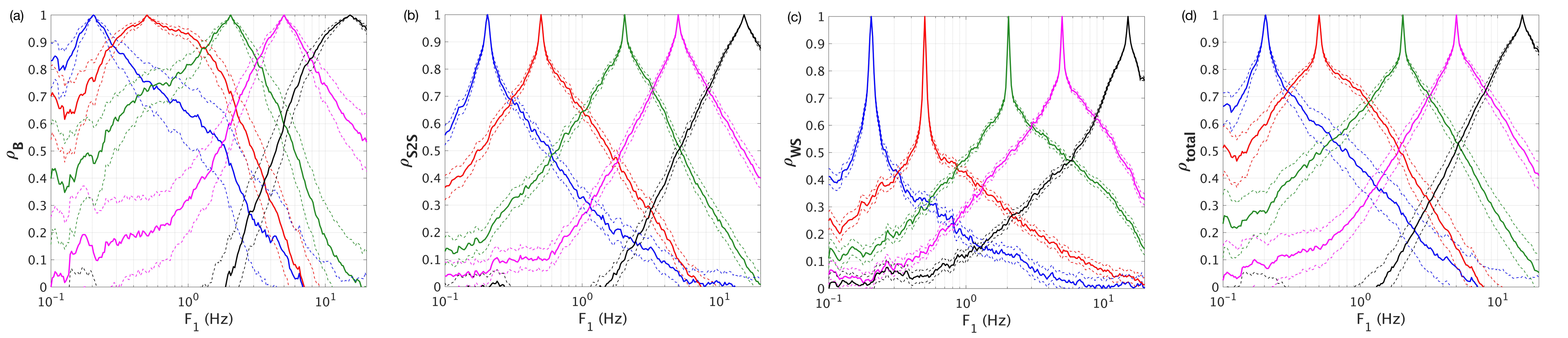

Figure 2: Empirical ρ_ϵ cross-sections versus frequency at conditioning frequencies 0.2, 0.5, 2, 5, and 15 Hz (solid lines), with 95% confidence bounds on ρ (dashed lines), for the same components as Figure 1.

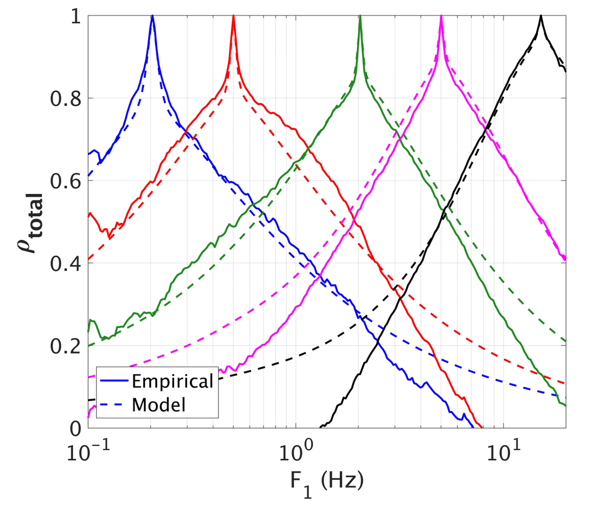

Figure 4: Model total ρ_ϵ cross-sections

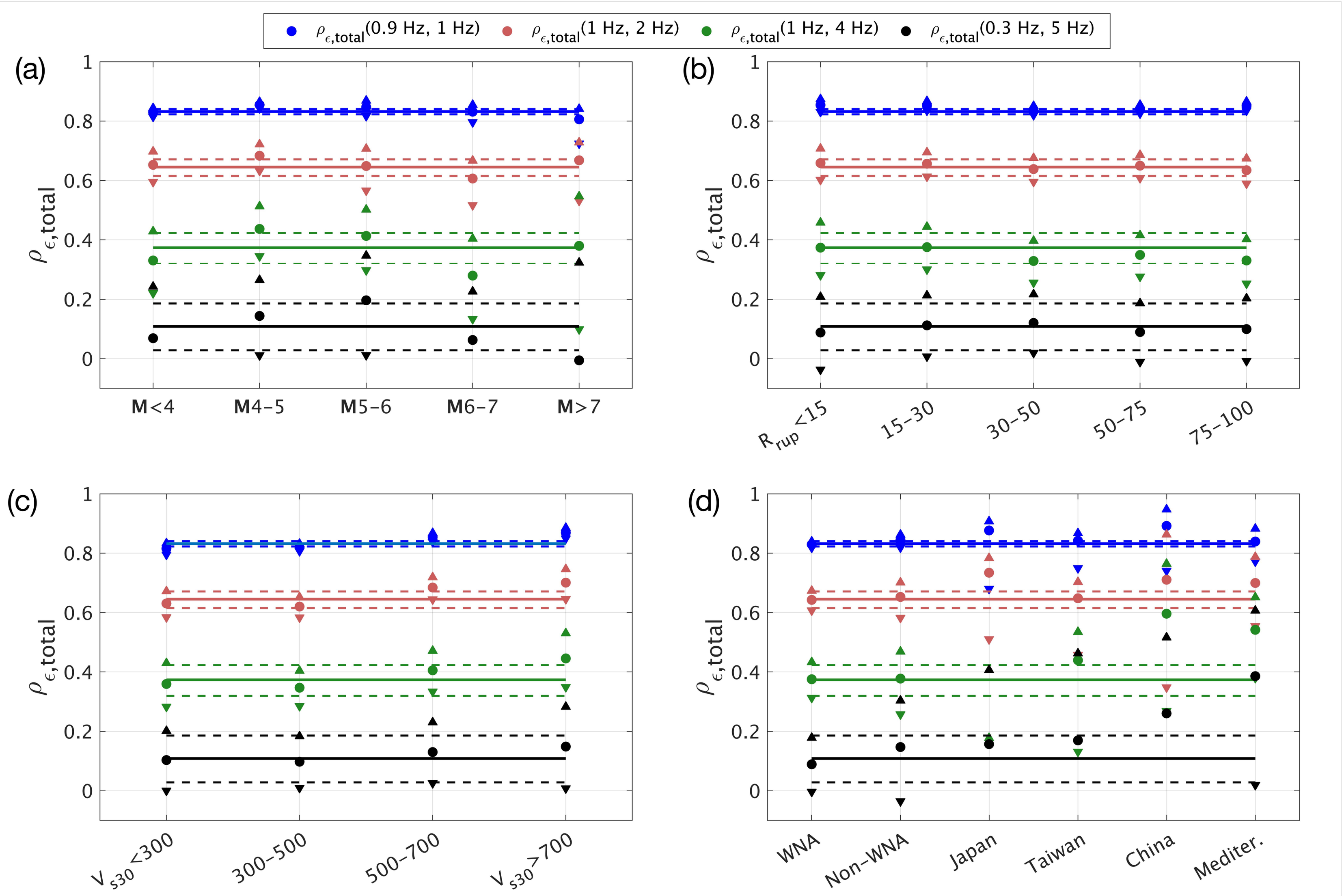

Figure 6: Evaluation of total ρ_ϵ for subsets of the data. The total ρ_ϵ for the full database is shown with the solid, horizontal lines, and dashed lines represent the lower and upper bounds for 95% confidence intervals of these coefficients (Kutner et al., 2004). The solid circles are the total ρ_ϵ calculated for each indicated data subset, and the triangles indicate 95% confidence intervals of those coefficients

(3) Conclusions

- The between-event residual standard deviation (τ) is larger than the other two standard deviation components at frequencies below 1 Hz, and above 1 Hz, the values of all three components are comparable (Figure 5). As a result, the between-event correlation contributes significantly to the total correlation, much more so than for response spectra. The between event ρ_ϵ physically relates to source effects (e.g. stress drop).
- This model exhibits higher correlation, especially at high frequencies, compared with the model of Stafford (2017), which did not smooth the Fourier spectra. It is expected the differences are primarily due to the distinct smoothing approaches.
- We do not find statistically significant magnitude, distance, site parameter, or regional dependence of the correlation, although potential regional variations should be studied further with more data from additional regions. Figure 6 summarizes the basis for this conclusion.

Model Range of Applicability

- Applicable to shallow crustal earthquakes in active tectonic regions worldwide, for rupture distances of 0 – 300 km, magnitude of 3.0 – 8.0, and for the frequency range 0.1 – 24 Hz.

Model Applications

- Can be used to define the inter-frequency correlation in stochastic ground-motion simulation methods such as Boore (2003). Stafford (2017) and Bayless and Abrahamson (2018) give examples of this procedure, using their respective correlation models, to modify the point-source stochastic simulation method to generate simulated acceleration time series with realistic ρ_ϵ.
- Calibration of the inter-frequency correlations from physics-based numerical simulations for ground motions from finite-fault earthquakes (Bayless and Abrahamson, 2018).
- Additional uses: conditional mean spectra for FAS, vector-valued PSHA for FAS.

Acknowledgement

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